

Interactions between Heterogeneity in Nominal Rigidities and Search Frictions in General Equilibrium Models

Submitted by Yongmin Park to the University of Exeter

as a thesis for the degree of

Doctor of Philosophy in Economics

In February 2018

This thesis is available for Library use on the understanding that it is copyright material and that no quotation from the thesis may be published without proper acknowledgement.

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature:

Abstract

This dissertation consists of three chapters that aim to build a framework which can be used to study interactions between the labour market and macroeconomic dynamics. To achieve this, we reformulate a standard New Keynesian dynamic stochastic general equilibrium (DSGE) model to include search and matching frictions in the labour market and heterogeneity in price and wage stickiness.

The first chapter, coauthored with Professor Engin Kara, builds a real business cycle model with labour search frictions and heterogeneity in wage stickiness. Shimer's (2005) critique on labour search models, that it cannot explain observed unemployment movements, reignited a long-standing debate on unemployment fluctuations and wage determination. Gertler and Trigari (2009) introduce wage stickiness to the model to match unemployment volatility, while Pissarides (2009) finds this modification not satisfactory, citing evidence on high wage cyclicality. We find heterogeneity in wage stickiness in microdata on wages. Our model, which reflects this heterogeneity, matches the data better than its one sector alternatives.

The second chapter, coauthored with Professor Engin Kara, studies output dynamics in New Keynesian models with the standard labour market and heterogeneity in price stickiness. We analytically and numerically show that these models can reproduce a hump-shaped output response to persistent monetary shocks, which is a key feature of monetary transmission mechanism. The version of models without heterogeneity cannot generate a hump. Flexible prices in models with heterogeneity play a crucial role, by generating inertia to price-setting and output.

The third chapter studies how the labour search frictions affect output dynamics in New Keynesian models, when combined with heterogeneity in nominal rigidities. Long-term employment relationship, that arises under search and matching framework, makes marginal costs history dependent. We show that this history dependence generates inertia in the model. Heterogeneity in nominal rigidities significantly reinforces this inertia, resulting in a hump-shaped output response to persistent monetary shocks. The model without the search frictions cannot replicate a hump even when monetary shocks are persistent, when wages are sticky.

Contents

Acknowledgements	7
Introduction	8
1 Unemployment volatility, the cyclicalities of wages, and heterogeneity in wage stickiness	15
1.1 Introduction	15
1.2 The model	21
1.2.1 The matching function	22
1.2.2 Firms	22
1.2.3 Households	24
1.2.4 Government and market clearing	26
1.2.5 Staggered wage bargaining in sector i	26
1.3 The log-linearised economy	30
1.3.1 Calibration	32
1.4 The volatility in labour market variables and the cyclicalities of wages	34
1.5 How does the heterogeneity generate higher wage elasticity?	38
1.5.1 High cyclicalities of wages and the flexible-wage sector	38
1.5.2 Long-term employment contract, wage elasticity, and labour market volatility	39
1.6 Bargaining set	40
1.7 Summary and Conclusions	42
2 Interactions between flexible prices and sticky prices lead to a hump-shaped output response to monetary policy	43

2.1	Introduction	43
2.2	The Multiple Calvo (MC) model	45
2.2.1	Choice of Parameters	47
2.3	A hump in output in the MC	47
2.4	Impulse response functions of output	52
2.5	The role of different parameter values	54
2.6	Summary and Conclusions	55
3	Output dynamics in New Keynesian models	
	with search frictions and heterogeneity in nominal rigidities	57
3.1	Introduction	57
3.2	The model	60
3.2.1	Overview of the Economy	60
3.2.2	The labour market and the matching	61
3.2.3	Households	62
3.2.4	Firms	63
3.2.5	Wage bargaining	64
3.2.6	Retailers and price setting	66
3.2.7	The rest of the model	67
3.3	Calibration	67
3.4	Impulse response functions	69
3.5	Discussion	72
3.5.1	Long-term relationship and history dependence	72
3.5.2	Long-term employment relationship and heterogeneity in price stickiness	75
3.6	Summary and conclusion	81
	Concluding remarks	83
	Appendices	85
	Bibliography	92

List of Figures

1.1	Distribution of wage spells in Barattieri, Basu, and Gottschalk (2014)	17
1.2	Impulse Response Functions of average wage to productivity shock . .	41
2.1	Flexible-price sector share and the coefficients on the output equation	50
2.2	Flexible-price sector share and the minimum shock persistence	51
2.3	Impulse Response Functions to monetary policy shock in 2-sector MC model	53
2.4	Impulse Response Functions to monetary policy shock in the BK-MC model	53
2.5	Flexible-price sector share and the minimum shock persistence, with different calibrations	55
3.1	Impulse Response Functions to monetary policy shock in models with and without the search frictions	70
3.2	Impulse Response Functions to monetary policy shock in models with the search frictions	71
3.3	Long-term labour contracts, the flexible-price sector share, and the coefficients on the output equation	78
3.4	Long-term labour contracts, the flexible-price sector share, and the minimum required shock persistence	80
3.5	An interaction between the long-term labour contracts and the flexible- price sector share on the minimum required shock persistence	81

List of Tables

1.1	Calibration	34
1.2	Main statistics	37
1.3	Semi-elasticity of wage to unemployment	38
3.1	Calibration	69

Acknowledgements

First of all, I would like to gratefully and sincerely thank my supervisor, Professor Engin Kara. Without his extraordinary guidance, understanding, and support during the last year of my graduate studies, this thesis would not have been possible. I learned not only from his expertise in economics, but also from his ceaseless passion for research. My second supervisor, Dr. Julian Neira Sanchez, helped me with his excellent guidance and kindness, so I thank him. I also would like to express my deepest gratitude to the Bank of Korea and University of Exeter, for financial support. I thank my colleagues and seminar participants at the University of Exeter Business School for their insightful comments.

Finally, with respect, I happily acknowledge the help of my loving wife, Joy Chang, whose love, patience, understanding, and most of all optimism gave me the strength to go on, when I needed it the most.

Introduction

The theory of the labour market has long been at the centre of economic theories, dating back at least to David Ricardo (1817, chapter 5). The predictions of macroeconomic theories mainly depend on how the labour market is modelled. For example, Keynes (1936) based his critique on classical economists, such as Pigou (1968), on their *strange supposition* about how wages and the labour supply and demand are determined. If our view on the labour market changes, our view on the macroeconomy changes, yielding different policy implications. This dissertation contributes to macroeconomic theories by developing a framework for a better understanding of the labour market and by exploring the implications of this framework for output dynamics, and more broadly for macroeconomy.

Our starting point is search and matching theory in the labour market. This theory, of which three inventors, Diamond (1982a,b), Mortensen (1982), and Pissarides (1985)¹, were awarded a Nobel Prize in economics in 2010, has become a standard tool in analysis of unemployment dynamics. As Shimer (2005) explains, this model is attractive for many reasons: it provides an appealing description of the labour market, and yet is analytically tractable; comparative statics are intuitive and rich; it can be adapted to answer a number of policy questions about the labour market.

Shimer (2005), however, also points out that the labour search theory cannot explain the cyclical behaviour of unemployment observed in U.S. data for a standard calibration (see also Costain and Reiter (2008)). This is called the unemployment volatility puzzle (Pissarides (2009)). A large body of literature emerged to take on this puzzle. One important approach to the problem, following Keynes (1936), proposes to solve the puzzle by introducing wage stickiness to the model (Shimer

¹See also Mortensen and Pissarides (1994).

(2005) and Hall (2005)). Gertler and Trigari (2009) carry this intuition to a general equilibrium Real Business Cycle (RBC) framework. They show that this model can explain key movements in labour market activity, including large volatility in unemployment. The intuition behind this result is that the response of wages to productivity shocks becomes muted, and firms create jobs aiming to capture a larger surplus from new jobs. Pissarides (2009) disagrees with this modification, arguing that wages are flexible at the relevant margin. Citing micro-evidence on high cyclicity of wages, he argues that wage stickiness cannot be an answer to the puzzle. This ongoing (and long-standing) debate gives rise to our first research question: how to explain observed volatility of unemployment and high wage cyclicity at the same time? Can wage stickiness be indeed an answer to the puzzle?

This question is particularly interesting, because it also poses an important challenge for monetary policy analysis. Modern monetary business-cycle models rely on price and wage stickiness to generate large real effects of monetary policy. For example, standard medium-sized New Keynesian general equilibrium models, such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), typically adopt wage stickiness. If wages are indeed flexible at the relevant margin, these models need to find an additional mechanism to explain the effects of monetary policy shocks. In this case, as Basu and House (2016) argue, price stickiness likely plays a substantially more important role than wage stickiness in explaining economic fluctuations.

In the first chapter, we build a model that can address this question. Then, in the second and the third chapters, we explore broader macroeconomic implications of our model in New Keynesian environment. In the second chapter, we first develop a novel approach to summarise simple New Keynesian models to one reduced-form output equation. Applying the method of undetermined coefficients to this equation reveals information on the degree of inertia in the model. In the third chapter, by using this approach, we find that our model shows a significant departure from models with the standard labour market, in terms of output dynamics. The specific aspects of output dynamics that we aim to match with our model is related to our

second research question.

The second research question of this dissertation concerns the real effects of monetary policy shocks. In particular, we focus on a hump-shaped response in aggregate spending to monetary shocks. The observation on monetary nonneutrality, i.e., that monetary policy has significant real effects in the short-run, dates back at least to David Hume (1752). Many modern empirical studies have explored the transmission mechanism of monetary policy. A key finding of these studies is that real activities demonstrate a hump-shaped and gradual response to monetary shocks (see, e.g., Sims (1986), Christiano, Eichenbaum, and Evans (1996, 1999), and Rotemberg and Woodford (1997)). As Christiano, Eichenbaum, and Trabandt (2017) state, a common prescription to this problem is to assume habit-formation in consumption. For example, widely-used medium-sized New Keynesian DSGE models such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) adopt this assumption. While we can find evidence from finance and growth literature that support the assumption (Fuhrer (2000) and Christiano, Eichenbaum, and Trabandt (2017)), there also have been studies that propose alternative mechanisms, arguing that this assumption remains ad-hoc and therefore subject to Lucas's (1976) critique.

In order to answer these two research questions, this dissertation pays attention to heterogeneity in price and wage stickiness. Bils and Klenow (2004) and Nakamura and Steinsson (2008) find that heterogeneity in price stickiness is an important feature of micro-evidence on prices. In recent years, many studies have explored the potential of this heterogeneity to help New Keynesian models in addressing important criticisms on them (see, for example, Carvalho (2006), Dixon and Kara (2010a), Dixon and Le Bihan (2012), and Kara (2015), among others). Based on this success, Taylor (2016) concludes that "future research would likely yield large benefits if it moved on from representative staggered wage and price setting models to heterogeneous staggered wage and price setting models."

Note that the labour search theory and heterogeneity in nominal rigidities have been studied extensively, but *separately*. We bring these two distinct studies together

and show that interactions between them can help us address two important challenges in macroeconomic modelling. To the best of our knowledge, this dissertation is the first attempt to do so.

In the first chapter, as mentioned, we take up the first challenge. We introduce heterogeneity in wage stickiness, based on micro-data on wage stickiness in US economy, provided by Barattieri, Basu, and Gottschalk (2014). The distribution of wage spells clearly indicates that there is heterogeneity in wage stickiness. We extend an otherwise standard search and matching model to reflect this heterogeneity. There are many sectors, each with a different degree of wage stickiness. In each sector, there is a Calvo-type contract in a Multiple Calvo (MC) model. Following the suggestion of Barattieri, Basu, and Gottschalk (2014) that the wage data favour Taylor-type contracts than Calvo-type contracts, we also consider a version of our model in which within each sector, there is a Taylor-style contract. Hence a Generalised Taylor Economy (GTE). In this way, we make methodological contribution to the literature by generalising the Calvo-type staggered bargaining, proposed by Gertler and Trigari (2009), to another important time-dependent contract structure, within search and matching framework.

We evaluate the extent to which our models can match both the volatility of labour market variables, as reported in Shimer (2005) and Gertler and Trigari (2009), and the cyclicalities of wages as reported in Pissarides (2009) and Haefke, Sonntag, and Van Rens (2013). Our results suggest that all four models generate similar level of volatility in labour market variables to that in the data. However, the models differ significantly about their predictions of the cyclicalities of wages. In the MC and the GTE, the semi-elasticity of wage to unemployment is twice as large as that in the Taylor model and is closer to that suggested by the data. We further find that the reason for higher elasticity in the GTE and in the MC is the presence of flexible wages. Since flexible wages respond more to shocks than sticky wages, average wages in the two models are more sensitive to shocks than their one-sector counterparts. While average wages in sectors with sticky wages do not change much in response to shocks, they do in the flexible sector, resulting in cyclical aggregate wages.

The second and the third chapters involve a hump-shaped output response to monetary shocks. The challenge is to generate inertia in output dynamics in an otherwise purely forward-looking model, as habit-formation does. Let us begin with the role of heterogeneity in price stickiness for output dynamics. In the second chapter, we present a simple New Keynesian model with the standard labour market. There are many sectors, each with a different contract length. We calibrate the model by using micro-data on price stickiness provided by Bils and Klenow (2004). We find that, when monetary policy shocks are persistent, an interaction between sectors with flexible prices and those with sticky prices leads to a hump-shaped output response.

The key to our results is the role played by flexible prices. We formally demonstrate how the presence of flexible prices makes price-setting less forward-looking. To understand this, first note that sectors with flexible prices respond more to shocks than the ones with sticky prices. Soon after the shock, to preserve relative prices and demand, flexible-price firms reduce their prices to bring them in line with those in sticky-price sectors. As a consequence, inflation in the flexible-price sector depends negatively on relative price in that sector in the previous period. This behaviour makes price-setting history dependent in the sense that inflation in the flexible-price sector and, in turn, aggregate inflation depend on lagged relative prices. Relative prices are related to output. Therefore, output in the previous period becomes relevant. This history dependence, in the presence of persistent monetary policy shocks, leads to a hump-shaped response in output.

For our analysis, we propose a novel method to summarise a simple New Keynesian model with and without heterogeneity in price stickiness to one reduced-form equation for output dynamics. By applying the method of undetermined coefficients to this equation, we show that output in the multi-sector model can be expressed as an autoregressive process, where output depends on its own lag and monetary policy shocks. A higher share of flexible prices implies a greater coefficient on lagged output, leading to a larger hump. The Calvo model, a special case of the multi-sector model when the share of the flexible-price sector is zero, cannot generate a hump,

as output lacks backward-looking behavior.

In the third chapter, we bring together our findings in the first and the second chapters. Our aim is to revisit the role of the search and matching frictions for output dynamics in New Keynesian business cycle models, in combination with heterogeneity in nominal rigidities. Many studies have introduced the search and matching frictions in the labour market in New Keynesian environment. The conclusions of these studies are mixed. For example, Walsh (2005) and Trigari (2009) study how the search and matching frictions affect the effects of monetary policy shocks. They find that the search frictions increase output responses relative to otherwise similar models with the standard labour market. In contrast, Krause and Lubik (2007) point out that the precise role of the search frictions in Walsh (2005) and Trigari (2009) are difficult to assess, due to model assumptions other than the search frictions such as habit formation. Based on an analysis of a simple New Keynesian model with the search frictions, they conclude that the search frictions *per se* do not affect the mechanism that generates endogenous persistence in the model.

Our main point of departure from these studies is that we introduce heterogeneity in price and wage stickiness. We build on the real business cycle model with the search and matching frictions and wage rigidity, that we developed in the first chapter. We bring this model into New Keynesian environment and introduce heterogeneity in price stickiness.

Our key finding is that long-term employment relationship that arises under search and matching framework makes marginal costs history dependent: marginal costs depend on lagged output level. Search and matching models explicitly stipulate that previously-employed workers stay in the job for a long period of time. In a model with the standard labour market, where hiring is costless, this would not matter. But with a costly and time-consuming hiring process, the (partial) continuation of previously created job matches implies that firms could save hiring costs in the current period thanks to past hiring activities, and hence, with other things equal, marginal costs would be lower if they hired more workers in the past. As a result, lagged employment, and lagged output level affect marginal costs in the current

period. Inflation depends on marginal costs, and on lagged output level. Therefore previous path of output becomes relevant.

By using the method that we developed in the second chapter, it is analytically shown that output exhibits an inertial response. The inertia becomes stronger when labour contracts are based on longer-term relationship. With our baseline calibration, this effect of the search frictions *alone* is modest. When combined with heterogeneity in nominal rigidities, however, this tendency is significantly magnified. On the other hand, we show that the potential of heterogeneity in price stickiness to generate a hump in output, which is explored in the second chapter, can be weakened if wages are sticky. This is because even flexible prices do not adjust to shocks due to sluggish adjustments of marginal costs. When we introduce the search frictions, the potential of heterogeneity in price stickiness is reinvigorated. We conclude that the search frictions and heterogeneity in nominal rigidities reinforce each other to yield a mechanism that generates inertia in the model. This results in a hump-shaped output response to monetary shocks.

Chapter 1

Unemployment volatility, the cyclicalities of wages and heterogeneity in wage stickiness

1.1 Introduction

The search and matching model proposed by Diamond (1982a,b), Mortensen (1982), and Pissarides (1985) (see also Mortensen and Pissarides (1994)) has been a standard tool in analysing unemployment dynamics. The model is rich enough to account for several important features of labour market dynamics. In the model, in each period, a certain fraction of workers lose their job and search for a new one. Aiming to capture the difference between workers' productivity and workers' wage, firms create new jobs. The model can account for the fact that in a tight market, firms find it difficult to hire new workers.

However, Shimer (2005) criticises the model on the grounds that it cannot generate sufficient volatility in labour market variables in response to an increase in productivity, calling into question the empirical relevance of the model. This result is a consequence of the model's assumption that wages are fully flexible. The reason for this result is that all the increase in productivity is absorbed by wages, leaving no incentive to firms to create jobs. As a result, in contrast to empirical findings, employment does not change much in the model.

Shimer's conclusion started a lively debate on how best to model wages in a search and matching model. The first strand of the literature, following Keynes

(1936), proposes to solve the puzzle by introducing wage stickiness to the model (Shimer (2005) and Hall (2005)). Gertler and Trigari (2009) carry this intuition to a general equilibrium Real Business Cycle (RBC) framework. They show that adding wage stickiness to the model leads to spillover effects, resulting in larger fluctuations in unemployment and in other labour market variables. Since wages are sticky, not all wages are adjusted to reflect the increased productivity. As a consequence, relative to the increased productivity, wages paid by some employers remain low, lowering workers' opportunity costs. For this reason, workers who negotiate their wages ask for lower wages than they otherwise would. Increased productivity and low wages induce firms to create jobs, reducing unemployment further.

Another strand of the literature, namely that of Pissarides (2009), disagrees with the idea of introducing wage stickiness to the model. Pissarides (2009) argues that the relevant wage for job creation is new hires' wages and the micro-data on wages suggest that these wages are significantly more volatile than existing workers' wage. Based on this evidence, he concludes that new hires' wages cannot be sticky. Therefore, wage stickiness cannot be the answer to the puzzle.

Recently, Gertler, Huckfeldt, and Trigari (2016) question the validity of the finding that new hires' wages are more cyclical than those of existing workers and, therefore, the notion that new hires' wages are flexible. They argue that the finding is a consequence of the common failure to account for the fact that during expansions some workers may be moving to higher-paying jobs. Their findings suggest that, once such job changes are accounted for in the estimation, the wages of newly hired workers are not significantly more cyclical than those of existing workers. Stuber (2017) and Hagedorn and Manovskii (2013) reach similar conclusions.

The finding of Gertler, Huckfeldt, and Trigari (2016), however, does not put the debate to rest. Empirical studies surveyed by Pissarides (2009) suggest that the wages of existing workers are highly cyclical, too. For example, findings reported in Haefke, Sonntag, and Van Rens (2013) indicate that one percentage point increase in the unemployment rate leads to a 1 – 1.5% decrease in real wages. If it is indeed the case that wage cyclicalities are a good measure of wage flexibility, the evidence

provided by Gertler, Huckfeldt, and Trigari (2016) does not provide a strong case against Pissarides’s argument that wages are flexible.

The finding of Gertler, Huckfeldt, and Trigari (2016) that new hires’ wages are the same as those of continuing workers is also important in that it opens the door to make use of the micro-evidence that are available for existing workers’ wages. While, to the best of our knowledge, the direct evidence of wage stickiness for new hires’ wages is not available, there is evidence for existing workers’ wages. In their online appendix, Barattieri, Basu, and Gottschalk (2014) provide evidence for the distribution of wage spells for the US economy. The data are based on the Survey of Income and Program Participation conducted by the US Census Bureau and are for the period from 1996 to 2000. The distribution of wage spells is reported in Figure 1.1. As the figure shows, even though the data are for existing workers, there are plenty of wages that are relatively flexible, providing support for Pissarides’s argument. The proportion of wages that change within a year is more than 50%. There are also quite a few long wage contracts. The distribution has a peak at one year. The average wage spells, which include incomplete durations, is around 6 quarters.

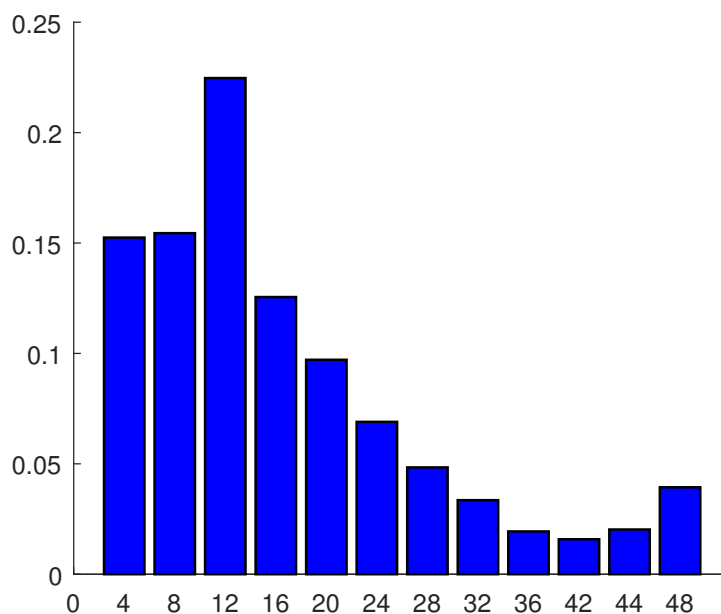


Figure 1.1: Distribution of wage spells in Barattieri, Basu, and Gottschalk (2014)

Note: The wage data was recorded every 4 months. Each bar denotes the corresponding share for each 4-months period.

The challenge is to develop a model that can match both the observed volatility of unemployment and the high cyclical of wages. This chapter takes this challenge. To achieve this, we introduce heterogeneity in wage stickiness suggested by micro-evidence on wages to an otherwise standard search and matching model. There are many sectors, each with a different degree of wage stickiness. To be more specific, we assume that there is a large number of firms and of households. Households have many members, who can be both unemployed and employed. We group firms according to the degree of wage stickiness they face. There are N groups (or sectors). As in the standard search and matching model, firms post vacancies and hire in a common labour market. Therefore, the only difference between our model and the standard model is the contract structure. Indeed, our model has the standard search and matching model and the model by Gertler and Trigari (2009), as special cases. When all sectors have flexible wages, we have the standard search and matching model. When all sectors face the same degree of wage stickiness, the model becomes the same as that in Gertler and Trigari (2009).

As it is common in the literature, we assume the probability of wage change is constant, as in the Calvo model and interpret each wage spell reported in Figure 1.1 as a Calvo wage reset probability. We use these probabilities to calibrate our model, resulting in a Multiple Calvo (MC) model. Barattieri, Basu, and Gottschalk (2014) suggest that the wage data favour Taylor-type contracts than Calvo-type contracts. Based on this finding, we also consider a version of our model in which within each sector, there is a Taylor-style contract, giving rise to a Generalised Taylor Economy (GTE). Using the wage spells data and the formula put forward by Dixon and Kara (2006), we calculate the distribution of completed durations. The resulting distribution is used to calibrate the GTE. Finally, we consider the versions of these models without heterogeneity: the simple Taylor and the Calvo models.

We evaluate the extent to which our model can match both the volatility of labour market variables, as reported in Shimer (2005) and Gertler and Trigari (2009), and the cyclical of wages as reported in Pissarides (2009) and Haefke, Sonntag, and Van Rens (2013). Our results suggest that all four models generate more or less the

same level of volatility in labour market variables. These volatilities are similar to those in the data. However, the models differ significantly about their predictions of the cyclicalities of wages. In the MC and GTE, average wage is significantly more responsive to productivity shocks than the Taylor and the Calvo models. In the Taylor model, the semi-elasticity of wages to unemployment is the lowest, since only a small fraction of wages adjust in each period. In the MC and the GTE, the semi-elasticity is twice as large as that in the Taylor model and is closer to that suggested by the data.

The reason for higher elasticity is the presence of flexible wages in the GTE and in the MC. Since flexible wages respond more to shocks than sticky wages, average wages in the two models are more sensitive to shocks than their one-sector counterparts. While average wages in sectors with sticky wages do not change much in response to shocks, they do in the flexible sector, resulting in cyclical aggregate wages.

There are many studies that provide theoretical justification for wage rigidity; Menzio and Moen (2010), Menzio (2005), and Kennan (2010) to name a few. Menzio and Moen (2010) is especially relevant for our work. Menzio and Moen (2010) provide a mechanism that justifies the finding of Gertler, Huckfeldt, and Trigari (2016) that new hires' and existing workers' wages have similar degrees of wage rigidity. They consider a labour market in which firms insure existing employees against income fluctuations. They show that, if firms can commit to a wage policy but not to employ workers, the optimal wage policy prescribes the same degree of downward wage rigidity for both existing workers and new hires.

Kudlyak (2014) argues that the relevant margin for firms' hiring decision is the user cost of labour, which may be different from the current wages. In a model like ours in which existing workers and new hires face the same degree of wage rigidity, the user cost of labour is given by current wages and the hiring margin by the present value of current and future wages over the contract spells. As a result, given that sectors face different wage stickiness, there is heterogeneity in the user cost of labour too.

In the New Keynesian literature, accounting for the heterogeneity in prices has proved to be helpful in addressing the criticisms directed at New Keynesian models (see Taylor (2016) for a survey). For example, Kara (2015) shows that two disturbing problems of the Smets and Wouters (2007) model, which is considered to be a state of art instance of New Keynesian economics, disappear when heterogeneity in price stickiness is introduced. First, the model requires large price shocks to explain inflation dynamics (see Chari, Kehoe, and McGrattan (2009)) and, second, firm level pricing in the model is inconsistent with that in reality (see Bils, Klenow, and Malin (2012)).

Another paper that emphasises the importance of heterogeneity in price stickiness is Carvalho (2006). Different from Kara (2015), in which it is assumed that there are sectors, each with a different contract length, Carvalho's (2006) model assumes that there are different industries. In the case of micro-data on prices (e.g. Bils and Klenow (2004)), it happens that different industries face different degrees of price stickiness. This is how heterogeneity in price stickiness is introduced to the Carvalho's (2006) model. However, wage data are different. Micro-data on wages provided by Barattieri, Basu, and Gottschalk (2014) suggest that there is little heterogeneity across industries, whereas there is a significant degree of heterogeneity in wage spells, providing empirical support for the modelling approach in Kara (2015).

The remainder of the chapter is organised as follows. Section 2 presents the model. Section 3 presents the log-linearised model and discusses the calibration of model parameters. Section 4 evaluates the empirical performance of the model and shows that models with heterogeneity in wage stickiness come closer in matching the data. Section 5 explains why models with heterogeneity in wage stickiness match the data better. Section 6 checks if the models pass the Barro's (1977) test. Section 7 concludes the chapter.

1.2 The model

We generalise the model of Gertler and Trigari (2009) to include many sectors, each with a different contract length. As noted above, we consider two alternative ways of modelling heterogeneity in wage stickiness: the GTE and the MC. There is a continuum of competitive firms with index $f \in [0, 1]$. The firms are divided into sectors with index i and sector shares are given by $\{\alpha_i\}_{i=1}^N$. Corresponding to the continuum of firms, there is a continuum of identical households of measure unity. Households supply labour. Firms hire workers from a common labour market. In the flexible-wage version of the model, once a worker finds a match with a firm, they negotiate a wage rate. In the MC and in the GTE, when negotiating, firms and workers take into account the fact that wages are sticky. In the MC, in each sector i , a randomly chosen fraction $1 - \delta_i$ of wages are negotiated in each period. In the GTE, sector i is divided into i cohorts: one cohort resets its wage in each period for i -periods. The standard Calvo and the simple Taylor models are a special case of the model, when all sectors face the same degree of wage stickiness¹.

The rest of model assumptions are standard. The government uses lump-sum tax to finance the unemployment benefit, which is paid to unemployed workers. A firm's workforce consists of workers that are employed in the past and new hires. It is assumed that firms lose a fraction λ of workers in each period. We denote the employment of firm f in sector i in period $t - 1$ as n_{fit-1} . After the separation, the number of existing workers in the firm f in period t is given by $(1 - \lambda)n_{fit-1}$. In each period, firm f posts vacancies v_{fit} to hire new workers $q(\theta_t)v_{fit}$ where $q(\theta_t)$ denotes the vacancy-filling rate of the firms, which is defined below. The hiring rate of the firm f is defined as $x_{fit} \equiv q(\theta_t)v_{fit}/n_{fit-1}$. Therefore, firm f 's employment evolves according to

$$n_{fit} = (1 - \lambda + x_{fit})n_{fit-1} \tag{1.1}$$

¹Due to the assumption of constant returns in matching, and since it is assumed that all workers have the same productivity, all workers are the same and set the same wage. In the firms that are not chosen to renegotiate their wage contracts, all existing workers and new hires get the same wage that is set in the past. Reset wages differ across sectors, since when firms and workers set their wages, they set them for different horizons.

This equation is based on the assumption that new hires participate in production immediately. The same assumption is made in Blanchard and Gali (2010) and Gertler, Sala, and Trigari (2008). In the rest of this section, we will outline the main building blocks of the model. We first present the structures of the model that are common to all models: the matching function, firms, household, the government, and the market clearing condition. We then discuss the staggered wage bargaining in the models.

1.2.1 The matching function

The search and matching process is standard and is done at the economy-wide level. That is, job seekers are free to move to any sector when they find a job opportunity, regardless of their previous employment history. Given these assumptions, the total number of successful matches in the economy in period t is given by the following matching function

$$m(u_t, v_t) \equiv \mu_m u_t^\mu v_t^{1-\mu}, \quad 0 < \mu < 1$$

where $v_t (\equiv \int_0^1 v_{fit} df)$ is the total number of vacancies posted by firms and u_t is the total number of job seekers (or unemployed workers) in the economy. μ and μ_m denote the unemployment elasticity and the scale parameter of the matching function, respectively. The total unemployment is given by $u_t = 1 - n_{t-1}$, since it is assumed that all unemployed workers search for jobs and the newly separated workers do not participate in searching in the same period. As is standard, we define the job finding rate of workers as $p(\theta_t) \equiv m(u_t, v_t)/u_t$, the vacancy filling rate of firms as $q(\theta_t) \equiv m(u_t, v_t)/v_t$, and the labour market tightness as $\theta_t \equiv v_t/u_t$.

1.2.2 Firms

There is a continuum of competitive firms. A firm produces a homogeneous consumption good. In each period, firm f employs n_{fit} workers to produce output y_{fit} . Each worker receives a wage w_{fit} . The production function with a constant returns

to scale technology is given by

$$y_{fit} = A_t n_{fit} \quad (1.2)$$

A_t denotes productivity which is assumed to follow an AR(1) process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (1.3)$$

where $a_t \equiv \log A_t$ and ϵ_t^a is an *iid* productivity shock with mean zero. The firm posts v_{fit} vacancies and hires $q(\theta_t)v_{fit}$ workers in period t . The hiring process is costly. Following Gertler and Trigari (2009), we assume that hiring costs take the following form

$$\frac{\kappa}{2} x_{fit}^2 n_{fit-1}$$

Taking into account exogenous job separations and newly created matches the law of motion of the employment stock in firm f is given by

$$n_{fit} = (1 - \lambda + x_{fit}) n_{fit-1} \quad (1.4)$$

In each period firm f chooses x_{fit} to maximise its value by taking the total number of its employees at the beginning of period t (n_{fit-1}) and the current and expected future path of wages as given. Specifically firms solve the following problem

$$F_{fit}(n_{fit-1}) = \max_{x_{fit}} \{ A_t n_{fit} - w_{fit} n_{fit} - \frac{\kappa}{2} x_{fit}^2 n_{fit-1} + E_t \beta_{t,t+1} F_{fit+1}(n_{fit}) \} \quad (1.5)$$

subject to $n_{fit} = (1 - \lambda + x_{fit}) n_{fit-1}$. $\beta_{t,t+1}$ is the stochastic discount factor between periods t and $t + 1$ and will be defined below. The solution to this maximisation problem results in the job creation condition of the firm, which is given by

$$\kappa x_{fit} = A_t - w_{fit} + E_t \beta_{t,t+1} \{ \frac{\kappa}{2} x_{fit+1}^2 + (1 - \lambda) \kappa x_{fit+1} \} \quad (1.6)$$

This equation shows that the hiring rate depends on the net marginal product of labour ($A_t - w_{fit}$), savings on hiring costs in the next period and the continuation value of the match.

We define $J_{fit}^F(w_{fit})$ as the firm's marginal surplus when it hires an additional worker at the wage rate w_{fit} . This is given by

$$J_{fit}^F(w_{fit}) = A_t - w_{fit} + E_t \beta_{t,t+1} \left\{ \frac{\kappa}{2} x_{fit+1}^2 + (1 - \lambda) J_{fit+1}^F(w_{fit+1}) \right\} \quad (1.7)$$

By comparing the last two equations, we note that

$$J_{fit}^F(w_{fit}) = \kappa x_{fit} \quad (1.8)$$

This equation requires that in equilibrium the value of an additional worker to be equalized with the costs of adding one more worker.

1.2.3 Households

As it is standard in this literature, we use the representative family setup proposed by Merz (1995). As noted earlier, there is a continuum of identical households. Each household has a continuum of members, which can either be workers or unemployed. While household members work in different sectors, they pool income together and get full risk sharing within the household. As a consequence of these assumptions, all household members consume the same amount. An unemployed member of the representative household receives unemployment benefit. The representative household holds bonds and is a shareholder of firms and receives dividends. Given these assumptions, the representative household's life-time utility and the corresponding budget constraint are given by

$$U_t = \max_{c, B} \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta U_{t+1}$$

where σ is the constant-relative-risk-aversion parameter and β is the subjective discount rate. The household's budget constraint is

$$c_t + B_t \leq (1 + r_{t-1})B_{t-1} + \int_0^1 n_{fit} w_{fit} df + b(1 - n_t) + \Pi_t - T_t$$

where r_{t-1} is the (real) interest rate between period $t - 1$ and t , B_{t-1} is holdings of one-period real bonds, w_{fit} is the wage rate in firm f in sector i , and Π_t denotes aggregate dividends from the firms. In addition, b is unemployment benefit measured in consumption units and T denotes taxes. The representative household maximises its utility subject to the budget constraint. The first order condition of this problem is given by

$$1 = \beta E_t \left\{ (1 + r_t) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \right\} \quad (1.9)$$

Since the probability to find a job is $p(\theta_t)$, the household's employment in firm f in sector i evolves according to

$$n_{fit} = (1 - \lambda)n_{fit-1} + p(\theta_t) \frac{v_{fit}}{v_t} u_t \quad (1.10)$$

where $p(\theta_t)v_{fit}/v_t$ is the probability of finding a job at firm f in sector i .

We define $J_{fit}^W(w_{fit})$ as the workers' surplus from a job, i.e., the marginal value of additional employment in firm f in sector i to the household in consumption units. This is given by

$$J_{fit}^W(w_{fit}) = w_{fit} - b - E_t \beta_{t,t+1} \{ p(\theta_{t+1}) J_{xt+1}^W - (1 - \lambda) J_{fit+1}^W(w_{fit+1}) \} \quad (1.11)$$

where

$$J_{xt+1}^W \equiv \int_0^1 \frac{v_{jit+1}}{v_{t+1}} J_{jit+1}^W(w_{jit+1}) dj$$

denotes the economy-wide average surplus of a worker who is newly hired in time $t + 1$. $\beta_{t,t+k} \equiv \beta c_{t+k}^{-\sigma} / c_t^{-\sigma}$ is the stochastic discount factor between periods t and $t + k$. The last two terms in Equation (1.11) come from Equation (1.10) and reflect the fact that an additional unit of employment at firm f results in $1 - \lambda$ unit of surviving match at the firm in the next period but that this comes at a cost for workers. Workers lose opportunities to find jobs elsewhere in the economy in the next period, as workers who are employed cannot search for jobs in the next period.

1.2.4 Government and market clearing

The resource constraint is given by

$$y_t = c_t + \frac{\kappa}{2} \int_0^1 x_{fit}^2 n_{fit-1} df \quad (1.12)$$

The government budget constraint is

$$b(1 - n_t) = T_t \quad (1.13)$$

The equation is based on the assumption that the government finances unemployment benefits with taxes.

1.2.5 Staggered wage bargaining in sector i

In this section, we describe the wage bargaining in the MC and the GTE models. In both models, in each sector i , only a fraction of firms negotiate their wages with their workers in each period. Newly hired workers are assumed to receive the same wage with the existing workers if they enter the firm between the contracts. When negotiating their wages, workers and firms take into account the fact that the wage set is going to be valid for some time. We begin by describing the standard search and matching with flexible wages and, then, present MC and GTE models.

Equilibrium wages with flexible wage bargaining

We denote the wage in sector i by w_{it} ². Firm and worker surpluses, $J_{it}^F(w_{it})$ and $J_{it}^W(w_{it})$, are given by

$$J_{it}^F(w_{it}) = A_t - w_{it} + E_t \beta_{t,t+1} \left\{ \frac{\kappa}{2} x_{it+1}^2 (w_{it+1}) + (1 - \lambda) J_{it+1}^F(w_{it+1}) \right\} \quad (1.14)$$

$$J_{it}^W(w_{it}) = w_{it} - b - E_t \beta_{t,t+1} \{ p(\theta_{t+1}) J_{it+1}^W - (1 - \lambda) J_{it+1}^W(w_{it+1}) \} \quad (1.15)$$

²Since all renegotiating firms in a given sector set the same wage, we drop the subscript f from now on. Note that, under flexible bargaining, all wages across sectors are the same, and therefore we do not need the sector subscript i either. We maintain it for comparison with the MC and the GTE.

The Nash bargaining involves choosing the wage rate w_{it} that maximises the product of worker and firm surpluses. The resulting sharing rule is

$$J_{it}^W(w_{it}) = \eta\{J_{it}^W(w_{it}) + J_{it}^F(w_{it})\} \quad (1.16)$$

where $\eta \in (0, 1)$ denotes workers' bargaining power. When we apply the sharing rule (Equation (1.16)) to the firm and worker surpluses (Equations (1.14) and (1.15)), we obtain the following expression for wages.

$$w_{it} = w_{it}^{FLEX} \quad (1.17)$$

where

$$w_{it}^{FLEX} = \eta(A_t + E_t\beta_{t,t+1}\frac{\kappa}{2}x_{it+1}^2) + (1 - \eta)\{b + E_t\beta_{t,t+1}p(\theta_{t+1})J_{xt+1}^W\} \quad (1.18)$$

w_{it}^{FLEX} denotes the wage rate that resulted from the Nash bargaining problem when wages are fully flexible. As in the standard search and matching model, Equation (1.18) implies that flex wage is a weighted average of the workers' contribution to the match and the workers' opportunity costs.

The wage bargaining in the MC model

In the MC model, a random fraction $1 - \delta_i$ of firms in sector i resets the wage rate with workers. We denote the reset wage in sector i by w_{it}^* . Then, the firm surplus of the resetting firm in sector i can be rewritten as

$$\begin{aligned} J_{it}^F(w_{it}^*) &= A_t - w_{it}^* + E_t\beta_{t,t+1}\{\delta_i\frac{\kappa}{2}x_{it+1}^2(w_{it}^*) + (1 - \delta_i)\frac{\kappa}{2}x_{it+1}^2(w_{it+1}^*)\} \\ &\quad + (1 - \lambda)E_t\beta_{t,t+1}\{\delta_i J_{it+1}^F(w_{it}^*) + (1 - \delta_i)J_{it+1}^F(w_{it+1}^*)\} \end{aligned} \quad (1.19)$$

Equation (1.19) adjusts (1.14) to reflect the possibility that the wages will remain fixed with probability δ_i , and, therefore, the hiring rate and the firm surplus will be different from those of the resetting firms in the next period.

Similarly the worker's surplus from the match can be expressed as

$$J_{it}^W(w_{it}^*) = w_{it}^* - \{b + E_t p(\theta_{t+1}) \beta_{t,t+1} J_{xt+1}^W\} \\ + (1 - \lambda) E_t \beta_{t,t+1} \{\delta_i J_{it+1}^W(w_{it}^*) + (1 - \delta_i) J_{it+1}^W(w_{it+1}^*)\} \quad (1.20)$$

The sharing rule is given by Equation (1.16)³. When we apply the sharing rule (Equation (1.16)) to the firm and worker surpluses (Equations (1.19) and (1.20)), we obtain the following expressions for reset wages.

$$w_{it}^* = E_t \sum_{k=0}^{\infty} \psi_{i,k}^{MC} w_{it+k}^{FLEX} \quad (1.21)$$

where

$$\psi_{i,k}^{MC} \equiv \frac{\{\beta(1 - \lambda)\delta_i\}^k \Lambda_{t,t+k}}{\sum_{j=0}^{\infty} \{\beta(1 - \lambda)\delta_i\}^j \Lambda_{t,t+j}} \quad (1.22)$$

and $\Lambda_{t,t+k} \equiv c_{t+k}^{-\sigma} / c_t^{-\sigma}$. This equation indicates that reset wages are a weighted average of current and future flex wages during the expected wage spell, discounted by the survival probability of wage (δ_i), along with the survival probability of job ($1 - \lambda$) and the subjective discount factor (β).

The average wage (w_{it}) in sector i is given by

$$w_{it} = \sum_{k=0}^{\infty} \phi_{i,k}^{MC} w_{it-k}^* \quad (1.23)$$

where

$$\phi_{i,k}^{MC} = (1 - \delta_i) \delta_i^k \quad (1.24)$$

Equation (1.23) shows that average wage is a weighted average of current and past reset wages.

³Following Thomas (2008), we exclude the horizon effect, which results from the fact that workers and firms have different horizons when negotiating wages. A firm takes into account the fact that new hires will receive the same wage too. On the other hand, for workers, the current wage rate is only relevant during the time they work for the firm. Gertler and Trigari (2009) report that, this effect is not significant. Therefore, for simplicity but without loss of significant generality, we ignore this effect.

The wage bargaining in the GTE

In the GTE model, a fraction $1/i$ of firms in sector i resets the wage rate. This wage will remain effective for and only for i periods. The firm surplus of the resetting firm in sector i can be rewritten as

$$J_{it}^F(w_{it}^*) = E_t \sum_{k=0}^{i-1} \{(1-\lambda)\beta\}^k \Lambda_{t,t+k} \{A_{t+k} - w_{it}^* + \beta_{t+k,t+k+1} \frac{\kappa}{2} x_{it+k+1}^2(w_{it}^*)\} + E_t \{(1-\lambda)\beta\}^i \Lambda_{t,t+i} J_{it+i}^F(w_{it+i}^*) \quad (1.25)$$

Similarly the worker surplus in the resetting firm in sector i is rewritten as

$$J_{it}^W(w_{it}^*) = E_t \sum_{k=0}^{i-1} \{(1-\lambda)\beta\}^k \Lambda_{t,t+k} [w_{it}^* - \{b + \beta_{t+k,t+k+1} p(\theta)_{t+k+1} J_{xt+k+1}^W\}] + E_t \{(1-\lambda)\beta\}^i \Lambda_{t,t+i} J_{it+i}^W(w_{it+i}^*) \quad (1.26)$$

Applying the sharing rule (Equation (1.16)) to firm and worker surpluses (Equations (1.25) and (1.26)) obtains the following expressions for reset wages.

$$w_{it}^* = E_t \sum_{k=0}^{i-1} \psi_{i,k}^{GTE} w_{it+k}^{FLEX} \quad (1.27)$$

where

$$\psi_{i,k}^{GTE} \equiv \frac{\{\beta(1-\lambda)\}^k \Lambda_{t,t+k}}{\sum_{j=0}^{i-1} \{\beta(1-\lambda)\}^j \Lambda_{t,t+j}} \quad (1.28)$$

As before, reset wage in the GTE is a weighted average of current and expected future flex wages during the contract length. Since there is no uncertainty about the opportunity of wage-renegotiation, the future flex wages are not discounted by the hazard rate (δ_i) as in the MC case.

Finally, the average wage in sector i evolves according to

$$w_{it} = \sum_{k=0}^{i-1} \phi_{i,k}^{GTE} w_{it-k}^* \quad (1.29)$$

where

$$\phi_{i,k}^{GTE} = \frac{1}{i} \quad (1.30)$$

As in the MC, average wages are a weighted average of all ongoing wages.

1.3 The log-linearised economy

In this section, we present the complete set of log-linearised equilibrium conditions. The steady-state of the model economy is presented in the Appendix. Variables with a hat are log deviations from the steady-state value and variables with a tilde the steady-state values. We begin by presenting the key equations describing wage dynamics and job creation. By log-linearising Equations (1.21) and (1.27), we obtain the following expressions for the reset wages in sector i

$$\hat{w}_{it}^* = \sum_{k=0}^S \psi_{i,k} \hat{w}_{it+k}^{FLEX} \quad \begin{cases} \text{MC: } S = \infty \\ \text{GTE: } S = i - 1 \end{cases} \quad (1.31)$$

where $\psi_{i,k}$ is given by the steady-state values of Equations (1.22) in the MC and (1.28) in the GTE, i.e. with $\tilde{\Lambda}_{t,t+k} = 1$. The log-linearised flex wage in sector i in case of the flexible wage bargaining (Equation (1.18)) is

$$\begin{aligned} \hat{w}_{it}^{FLEX} = & \eta(\varphi_a \hat{a}_t + \varphi_x E_t \hat{x}_{it+1}) + \varphi_\Lambda \hat{\Lambda}_{t,t+1} \\ & + (1 - \eta) \varphi_\theta E_t \{(1 - \mu) \hat{\theta}_{t+1} + \hat{j}_{xt+1}^W\} \end{aligned} \quad (1.32)$$

where $\varphi_a \equiv \tilde{A}/\tilde{w}$, $\varphi_x \equiv \beta \kappa \tilde{x}^2/(\tilde{w})$, $\varphi_\theta \equiv p(\tilde{\theta})\beta \tilde{j}^W/\tilde{w}$, and $\varphi_\Lambda \equiv \eta \varphi_x/2 + (1 - \eta) \varphi_\theta$. The log-linearised average wage in sector i (Equations (1.23) and (1.29)) is

$$\hat{w}_{it} = \sum_{k=0}^S \phi_{i,k} \hat{w}_{it-k}^* \quad \begin{cases} \text{MC: } S = \infty \\ \text{GTE: } S = i - 1 \end{cases} \quad (1.33)$$

where $\phi_{i,k}$ is given by Equations (1.24) in the MC and (1.30) in the GTE.

Next, when we log-linearise the job creation condition (Equation (1.6)), we obtain the hiring rate in sector i

$$\hat{x}_{it} = \varkappa_a \hat{a}_t - \varkappa_w \hat{w}_{it} + \varkappa_\Lambda E_t \hat{\Lambda}_{t,t+1} + \beta E_t \hat{x}_{it+1} \quad (1.34)$$

where $\varkappa_a \equiv \tilde{A}/\tilde{j}^F$, $\varkappa_w \equiv \tilde{w}/\tilde{j}^F$ and $\varkappa_\Lambda \equiv \beta(1 - \lambda/2)$. Iterating this equation forward suggests that the hiring rate depends on the current and future productivity net of wage. Note that due to long-term nature of contracts, firms consider not only current wage and productivity, but the expected present value of wages and productivity.

Log-linearising unemployment ($u_t = 1 - n_{t-1}$), labour market tightness ($\theta_t = v_t/u_t$) and the sector- i hiring rate ($x_{it} = q(\theta_t)v_{it}/n_{it-1}$) gives

$$\hat{u}_t = -\frac{p(\tilde{\theta})}{\lambda}\hat{n}_{t-1} \quad (1.35)$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \quad (1.36)$$

$$\hat{x}_{it} = -\mu\hat{\theta}_t + \hat{v}_{it} - \hat{n}_{it-1} \quad (1.37)$$

Employment in sector i (Equation (1.4)) is log-linearised as

$$\hat{n}_{it} = \hat{n}_{it-1} + \lambda\hat{x}_{it} \quad (1.38)$$

Aggregating individual firm's production function (Equation (1.2)) across firms in sector i and then log-linearising the resulting equation yield the sectoral output.

$$\hat{y}_{it} = \hat{a}_t + \hat{n}_{it} \quad (1.39)$$

The log-linearised version of the Euler equation (Equation (1.9)) is given by

$$\sigma(\hat{c}_t - E_t\hat{c}_{t+1}) + E_tr_{t+1} = 0 \quad (1.40)$$

As noted above (Equation (1.3)), the productivity shock follows an AR(1) process.

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \epsilon_t^a \quad (1.41)$$

Finally, we aggregate sectoral output, wage, employment, vacancy, and hiring rate to obtain economy-wide output (\hat{y}_t), wage (\hat{w}_t), employment (\hat{n}_t), vacancy (\hat{v}_t), and hiring rate (\hat{x}_t). For a variable z_t , the economy-wide aggregate \hat{z}_t is given by a weighted average of sectoral aggregate \hat{z}_{it} with the sector share α_i as the weights as shown by the following equation.

$$\hat{z}_t = \sum_{i=1}^N \alpha_i \hat{z}_{it} \quad (1.42)$$

The resource constraint closes the model.

$$\hat{y}_t = \tilde{y}_c \hat{c}_t + (1 - \tilde{y}_c) (2\hat{x}_t + \hat{n}_{t-1}) \quad (1.43)$$

where $\tilde{y}_c \equiv \tilde{c}/\tilde{y}$ denotes consumption share in output.

1.3.1 Calibration

We begin with the distribution of wage stickiness. We calibrate the distribution of contract durations in our model using data compiled by Barattieri, Basu, and Gottschalk (2014). Barattieri, Basu, and Gottschalk (2014) provide evidence for the distribution of wage spells using the Survey of Income and Program Participation conducted by the US Census Bureau. The data are for the period from 1996 to 2000. The distribution of wage spells is reported in Figure 1.1. It is clear that there are some flexible-wage contracts and at the same time, that the wage distribution has a long tail. The interviews for the survey are conducted every 4 months and the wage spell distribution is reported on the same basis. Therefore we use quarterly calibration to minimise potential bias from using higher-frequency calibration.

To be more specific, the distribution reported in Barattieri, Basu, and Gottschalk (2014) is an age distribution in the sense that it includes both complete and incomplete durations. As it is common in the literature, we assume that the probability of a wage change is constant and interpret the statistics reported by Barattieri, Basu, and Gottschalk (2014) as Calvo reset probabilities. We then use them to calibrate the MC. The mean hazard rate is 0.18. We use this number to calibrate the Calvo model. To calibrate the GTE we generate the distribution of completed durations within each sector, and aggregate across sectors to obtain the distribution in the economy as suggested by Dixon and Kara (2006). Finally, for computational purpose we truncate the GTE at $N = 36$ quarters. The proportion of contracts that last longer than 36 quarters is less than 4%⁴. The mean completed contract lengths is 10 quarters. We use this value to calibrate the simple Taylor model.

⁴Increasing the truncation point to 48 quarters reduces the proportion to less than 2%, but our main conclusion appears robust.

Next we present parameters that are common, which are standard (see, for example, Gertler and Trigari (2009) and Krause and Lubik (2007)). The discount factor is set to $\beta = 0.99$, the persistence parameter of the productivity shock is set to $\rho_z = 0.95$ and the constant-relative-risk-aversion parameter is assumed to be $\sigma = 1$. These are all standard values in the RBC literature.

Turning to the parameter values that are specific to the search and matching model, the separation rate is calibrated at $\lambda = 0.10$, which is based on the evidence that jobs last about two years and a half. We set the steady-state unemployment rate to $\tilde{u} = 0.12$. This is to allow for the potential job seekers such as discouraged workers, since our model does not include the labour-market participation decision. The implied job finding rate is $p(\tilde{\theta}) = 0.73$. Following Krause and Lubik (2007) and Den Haan, Ramey, and Watson (2000), we set the firm's vacancy-filling rate to $q(\tilde{\theta}) = 0.7$. The matching elasticity is calibrated at $\mu = 0.5$, while workers' bargaining power is set to $\eta = 0.5$. The unemployment benefit ratio \tilde{b} is the ratio of the unemployment flow value (b) to the steady-state flow contribution of the worker to the match ($\tilde{A} + \kappa/2\tilde{x}^2$). κ is the hiring cost parameter. κ and b are chosen in a way so that \tilde{b} is equal to 0.4. This requires setting $\kappa = 6.56$ and $b = 0.41$. The implied replacement ratio is $b/\tilde{w} = 0.43$ and the steady-state hiring costs to output ratio is $\frac{\kappa}{2}\tilde{x}^2\frac{\tilde{n}}{\tilde{y}} = 0.03^5$.

Finally, we check the robustness of our results to the changes in key parameter values: the steady-state job finding rate ($p(\tilde{\theta})$) and the unemployment benefit ratio (\tilde{b}). Our benchmark value for $p(\tilde{\theta})$ is at the middle of the values used in related literature. For example, Gertler, Sala, and Trigari (2008) use 0.95 while Den Haan, Ramey, and Watson (2000) use 0.45. For \tilde{b} , we use the value used in Gertler and Trigari (2009) and Shimer (2005). But Hall (2008) suggests 0.7, based on a broader definition of \tilde{b} as including utility from leisure. Therefore, we test the values of $p(\tilde{\theta})$ between 0.45 and 0.95, and \tilde{b} between 0.4 and 0.7. Our main conclusions do not change significantly.

⁵This value is higher than that assumed in Gertler and Trigari (2009). This is because in their model production function consists of both capital and labour. In our model labour is the only input. We can adjust \tilde{b} to calibrate $\frac{\kappa}{2}\tilde{x}^2\frac{\tilde{n}}{\tilde{y}} = 0.01$, as in Gertler and Trigari (2009). Our main conclusions are robust.

Table 1.1: Calibration

Subjective discount factor	β	0.99
Productivity autoregressive parameter	ρ_a	0.95
Productivity standard deviation	σ_a	0.0075
Separation rate	λ	0.10
Elasticity of matching to unemployment	μ	0.5
Worker's bargaining power	η	0.5
Unemployment rate	\tilde{u}	0.12
Unemployment flow value	b	0.4081
Labour adjustment cost parameter	κ	6.56

Note: Parameters in the rows 1-7 are fix. The rest of the parameters, i.e. the unemployment flow value (b) and the labour adjustment cost parameter (κ) are implied.

1.4 The volatility in labour market variables and the cyclicalities of wages

In this section, we evaluate the potential of our model in matching the volatility of labour market variables and the cyclicalities of wages. Table 1.2 reports the simulated moments of labour market variables and output from our models along with the corresponding moments from the US data. The data are taken from Gertler and Trigari (2009) and for the periods between 1964:Q1 and 2005:Q1. Table 1.3 reports the semi-elasticity of wages to unemployment from our model simulation and from the US data. The data are taken from Haefke, Sonntag, and Van Rens (2013). Panel A of Table 1.2 and the first row of Table 1.3 report the US data. The volatilities of unemployment, vacancy, and labour market tightness are significantly greater than that of output⁶, and therefore the volatilities of those variables relative to output are 5.15, 6.30, and 11.28, respectively. The point-estimates for the semi-elasticity of wage to unemployment range between -1.0 and -1.5.

⁶As in Gertler and Trigari (2009), the standard deviations of variables are expressed relative to the standard deviation of output.

Before reporting statistics from the models with wage stickiness, it is useful to discuss the special case of our model when wages are fully flexible, as in the standard search and matching model. The statistics from the flexible-wage version of the model are reported in Panel B of Table 1.2 and in the second row of Table 1.3. Consistent with the findings reported in Shimer (2005), when wages are flexible, the model cannot generate enough (relative) volatility in labour market dynamics to match the data. The volatility of unemployment in the model is only 1.04, while it is 5.15 in the data. The volatility of vacancy implied by the model is much lower than that in the data (1.54 in the model vs. 6.30 in the data). This is also true for the labour market tightness (2.40 in the model vs. 11.28 in the data). On the other hand, the semi-elasticity of wage is higher than the point estimates (-3.5 vs. -1.0 – -1.5).

Panels C of Table 1.2 and the third and the fourth rows of Table 1.3 report statistics from the models with sticky wages and without heterogeneity: the Calvo and the Taylor models. As is evident, both models come closer in matching the volatility of labour market variables. In the Calvo model, the volatilities of unemployment, vacancy, and tightness are 3.92, 5.71, and 9.04, compared to 1.04, 1.54, and 2.40 in the standard search and matching model. In the simple Taylor model, these are 4.01, 5.95, and 9.28. However, increased volatility comes at a significant cost. The wage cyclicality implied by the model is too low, compared to the data. The semi-elasticity of wage to unemployment is -0.38 in the simple Taylor model and is only slightly higher in the Calvo model at -0.53.

We now turn to the models with heterogeneity: the MC and the GTE models. Panels E and F of Table 1.2 and the fifth and the sixth rows of Table 1.3 report the statistics for the models. In the MC model, the volatilities of unemployment, vacancy, and tightness are 3.58, 5.26, and 8.27, respectively. In the GTE, those are 3.43, 5.09, and 7.94. These numbers suggest that the models with heterogeneity generate similar volatilities in labour market variables as the models without. On the other hand, the MC and the GTE generate significantly higher wage elasticity than the Calvo and the Taylor models. The semi-elasticity of wage to unemployment

of the GTE is -0.80. The semi-elasticity of wage of the MC is -0.70.

Finally, it is useful to note that all models generate more or less the same degree of persistence in labour market variables, as measured by first autocorrelations. The autocorrelations are reported in the second row of each panel. In, for example, the GTE, the autocorrelations of unemployment, vacancy, and tightness are 0.93, 0.84, and 0.92, while they are 0.93, 0.86, and 0.92 in the simple Taylor, compared to 0.91, 0.91, and 0.91 in the US data.

Table 1.2: Main statistics

	y	w	n	u	v	θ
A. U.S. Economy, 1964:1-2005:1						
Relative s.d.	1.00	0.52	0.60	5.15	6.30	11.28
Autocorrelation	0.87	0.91	0.94	0.91	0.91	0.91
B. Standard search and matching (Flexible wages)						
Relative s.d.	1.00	0.84	0.14	1.04	1.54	2.40
Autocorrelation	0.74	0.74	0.93	0.93	0.84	0.92
C. Calvo						
Relative s.d.	1.00	0.31	0.53	3.92	5.71	9.04
Autocorrelation	0.85	0.96	0.94	0.94	0.87	0.93
D. Simple Taylor						
Relative s.d.	1.00	0.35	0.55	4.01	5.95	9.28
Autocorrelation	0.85	0.96	0.93	0.93	0.86	0.92
E. MC						
Relative s.d.	1.00	0.33	0.49	3.58	5.26	8.27
Autocorrelation	0.84	0.95	0.93	0.93	0.85	0.92
F. GTE						
Relative s.d.	1.00	0.35	0.47	3.43	5.09	7.94
Autocorrelation	0.83	0.95	0.93	0.93	0.84	0.92

Note: Panel A reports the statistics for the U.S. economy for the periods between 1964:Q1-2005:Q1, which are taken from Gertler and Trigari (2009). Panels B-F report the statistics for models that are computed by simulating the model 500 times for 300 periods conditional on productivity shock. Changes in the number of simulations do not change the results. The statistics are averages over the HP-filtered simulations with smoothing parameter 1,600. The standard deviations (s.d.) of all variables are relative to output.

Table 1.3: Semi-elasticity of wage to unemployment

	Semi-elasticity
US data	-1 – -1.5
Model	
Flexible wages	-3.50
Calvo	-0.53
Simple Taylor	-0.38
MC	-0.70
GTE	-0.80

Note: The US data is taken from Haefke, Sonntag, and Van Rens (2013).

1.5 How does the heterogeneity generate higher wage elasticity?

The results in the previous section suggest that the models with heterogeneity come closer in matching both the volatilities of labour market variables and the elasticity of wages in the data than the models without. In this section, we explain the intuition behind these results. There are two reasons why the MC and the GTE perform better. The first reason is the presence of flexible wages in the model. The second reason is the long-term nature of employment contracts. We will explain each reason in turn.

1.5.1 High cyclicalty of wages and the flexible-wage sector

The reason why wages are more responsive to shocks in the models with heterogeneity is the presence of flexible wages. In models with heterogeneity, the fraction of firms that reset wages in a given period t is always greater than that in the models without. Therefore in the models with heterogeneity, average wage is more responsive to shocks than in the corresponding models without heterogeneity, resulting in

a larger wage elasticity.

To compare the fraction of firms that reset the wage rate in a given period t in different models, recall that it is given by a strictly convex function $f(x) = 1/x$, where x is the contract duration T in the simple Taylor and the age of contract A in the Calvo model. The fraction is given by $\sum_{i=1}^N \alpha_i f(x_i)$ in the GTE and the MC, where x_i is the contract duration in sector i , T_i in the GTE, and the age in sector i , A_i in the MC. In both cases, $\sum_{i=1}^N \alpha_i x_i = x$. Therefore, by Jensen's inequality, $\sum_{i=1}^N \alpha_i f(x_i) > f(x)$. The fraction of resetting firms in the model with heterogeneity is always greater than that in the models without. Average wage in the GTE and the MC is more responsive to shocks than the corresponding simple Taylor and the Calvo model in the initial periods, resulting in a larger wage elasticity.

1.5.2 Long-term employment contract, wage elasticity, and labour market volatility

To understand why the models with and without heterogeneity generate similar volatility of labour market variables, it is useful to consider the aggregate job creation conditions in the models. If we aggregate Equation (1.34) across sectors, and then iterate the equation forward, we obtain the following aggregate hiring rate.

$$\hat{x}_t = E_t \sum_{k=0}^{\infty} \beta^k \{ \varkappa_a \hat{a}_{t+k} - \varkappa_w \hat{w}_{t+k} + \varkappa_{\Lambda} \hat{\Lambda}_{t,t+k} \} \quad (1.44)$$

This equation illustrates a well-known implication of the search and matching framework for job creation: firms' hiring decision depends on the expected present value of productivity and wages rather than current wages, as is pointed out by, for example, Haefke, Sonntag, and Van Rens (2013). The reason why firms consider future wages when they create a job is because of the long-term nature of employment contracts. Since firms and workers maintain employment relationship until the match is terminated, if future wages are high (low), the present value of the profits from the job are expected to be low (high), leading to smaller (greater) job creation. As

a result of the long-term contracts, wages in the far future⁷ are important for job creation, as much as the current wages.

An important implication of Equation (1.44) is that we need to examine the entire path of wage response to productivity shocks to see the effects of heterogeneity on volatilities of labour market variables. Figure 1.2 plots the IRF of average wages to the productivity shock in the GTE and in the simple Taylor model. In the GTE, while average wage responds more to productivity shocks in the initial part of the wage adjustment process, the later part is dominated by longer-term wage contracts. Therefore, some time after the shock, wage adjustment in the GTE slows down. In the simple Taylor model, although the initial adjustment in wages is slow due to reason explained in the previous subsection, the renewal of all wages are completed earlier than in the GTE. This is because there are less longer-term wage contracts in the simple Taylor model. Therefore the peak wage level in the simple Taylor model is higher than that in the GTE. After that, the only deviation from the steady-state wages will be generated by what remains in the highly persistent productivity shock process. The same mechanism holds true in the MC and the Calvo models case.

Overall, Figure 1.2 suggests that the heterogeneity in wage stickiness, or more broadly, the way the wage stickiness is modelled can significantly change the *shape* of the wage path, but not the present value of current and future wages. As a result, job creation, and therefore the volatilities of labour market variables are more or less the same across alternative models, while the elasticities are quite different.

1.6 Bargaining set

In this section we show that our model is not subject to the criticism by Barro (1977). If a wage rate is set for a very long time, then, after some time, it may fall out of the bargaining set. This implies that some wages are so outdated that they do not reflect the current economic conditions. If this happens, either firms or workers may find it inefficient to maintain the labour contracts, since the wage rate

⁷Note that future wages are discounted by β , which means even distant future wages are given considerable weights.

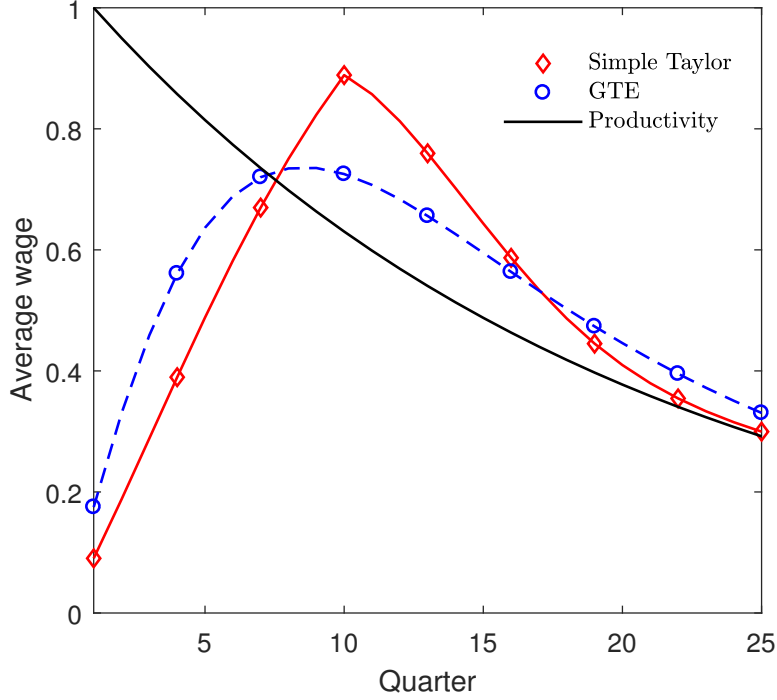


Figure 1.2: Impulse Response Functions of average wage to productivity shock

Note: The figure reports the IRF of average wage to productivity shock in the GTE and in the corresponding simple Taylor model.

is less than their opportunity costs. The lower bound of the bargaining set is given by the reservation wage of workers (r_{fit}^W), while the upper bound is determined by the maximum wage the firm is willing to pay (r_{fit}^F). In particular, the bargaining set is given by

$$B_{fit} = [r_{fit}^W, r_{fit}^F]$$

where r_{fit}^W is the wage at which the worker surplus from the job is zero,

$$J_{fit}^W = r_{fit}^W - b - E_t \beta_{t,t+1} \{p(\theta_{t+1}) J_{xt+1}^W - (1 - \lambda) J_{fit+1}^W\} = 0$$

and r_{fit}^F is the wage that makes the firm surplus from the job zero.

$$J_{fit}^F = A_t - r_{fit}^F + E_t \beta_{t,t+1} \left\{ \frac{\kappa}{2} x_{fit+1}^2 + (1 - \lambda) J_{fit+1}^F \right\} = 0$$

We test if contract wages stay within the bargaining set over the life of the contract. To check this, 1,000 observations of the model economy are simulated. Productivity shocks are assumed to be normally distributed with zero mean. We draw on standard values from the real business cycle literature and set the standard

deviation of the shock to 0.0075. This is also the value used in Gertler and Trigari (2009). For each τ -quarter-old wage, we compute the corresponding bargaining set and examine if the wage rate gets outside of the bargaining set. We then aggregate the fraction of contracts that can become inefficient, considering the share of the wage contracts in the economy. Our results show that about 95% of wage contracts remain efficient in the MC model and the GTE. All contracts are efficient in the simple Taylor model and the Calvo model.

1.7 Summary and Conclusions

We have extended the staggered multi-period wage contracting model of Gertler and Trigari (2009), which is based on the Diamond-Mortensen-Pissarides (DMP) framework to include many sectors, each with different degree of wage stickiness. Within each sector, there is a more or less standard search and matching process. When all sectors have the same degree of wage stickiness, the model reduces to the Gertler and Trigari (2009). Assuming in all sectors wages adjust every period gives the Diamond-Mortensen-Pissarides model.

Our main finding is that models that account for heterogeneity in wage stickiness suggested by micro-evidence on wages come closer in matching both the volatility of labour market variables and the cyclicalities of wages. We consider several different approaches in modelling wage stickiness. Results suggest that so long as there is some degree of wage stickiness, the model matches the volatility of labour market variables. However, only models with heterogeneity in wage stickiness can match both the volatility of labour market variables and the cyclicalities of wages. The presence of flexible wages in the economy means that such wages respond a lot to shocks, making aggregate wages more responsive to shocks. The presence of sticky wages means that it takes time for aggregate wages to adjust to shocks fully. Lower wages increase volatility in the labour market.

Heterogeneity in wage stickiness can offer a solution to the unemployment volatility puzzle and suggests a way forward in terms of how wages should be modelled in search and matching models.

Chapter 2

Interactions

between flexible prices and sticky prices
lead to a hump-shaped output response
to monetary policy

2.1 Introduction

The New Keynesian literature has struggled to come up with models that generate a hump-shaped output response to monetary policy shocks. While empirical studies show that monetary policy shocks lead to a hump-shaped response in output (see, for example, Christiano, Eichenbaum, and Evans (2005)), the standard simple New Keynesian model fails to generate a hump-shaped response. It has been difficult to identify mechanisms that can generate such a response. A common device to overcome this shortcoming of the model is to introduce habit persistence in consumption to the model (see, for example, Smets and Wouters (2007)). Doing so introduces inertia into an otherwise completely forward-looking model. If the degree of habit persistence is sufficiently large, the model generates a hump-shaped response in output in response to monetary policy shocks.

In this chapter, we explore another possible explanation for a hump-shaped response in output. We show that New Keynesian models that account for heterogeneity in price stickiness we have observed in the data can generate a hump-shaped response in output to monetary policy shocks. In the multi-sector model, there are

many sectors, each with a different contract length. We find that, when monetary policy shocks are persistent, an interaction between sectors with flexible prices and those with sticky prices leads to a hump-shaped output response.

Heterogeneity in price stickiness is an important feature of the micro-evidence on prices (see Bils and Klenow (2004) and Nakamura and Steinsson (2008)). Models that introduce this heterogeneity into New Keynesian models have been studied extensively in the recent literature (see, for example, Carvalho (2006), Dixon and Kara (2010), Dixon and Le Bihan (2012) and Kara (2015)). After reviewing these models, Taylor (2016) concludes that “future research would likely yield large benefits if it moved on from representative staggered wage and price setting models to heterogeneous staggered wage and price setting models”.

To the best of our knowledge, our work is the first to formalise the role of flexible prices on output dynamics in such models and to show how the presence of flexible prices can help the model generate a hump-shaped output response to monetary policy shocks. The existing literature tends to focus on macroeconomic persistence implications of these models.

Interestingly, our analysis reveals that adding flexible prices to an otherwise standard New Keynesian model makes price-setting less forward-looking. Indeed, using the undetermined coefficient method, we show that output in the multi-sector model can be expressed as an autoregressive process, where output depends on its own lag and monetary policy shocks. The lagged output term arises due to the introduction of a flexible-price sector to the model. The coefficient of lagged output depends on the share of the flexible-price sector and increases with the share of the flexible-price sector. We find that, holding other factors constant, a higher share of flexible prices leads to a larger hump. We further show that the Calvo model is a special case of the multi-sector model when the share of the flexible-price sector is zero and all the other sectors face the same degree of price stickiness. The Calvo model cannot generate a hump, as output does not have a lag and solely depends on the shock. This finding is consistent with that reported in Galí (2015, p. 64-66). Due to the lack of backward-looking behaviour in price-setting, the model cannot

generate a hump even when the shock is highly persistent.

To understand the reasons behind these results, first, note that sectors with flexible prices respond more to shocks than the ones with sticky prices. Soon after the shock, flexible-price firms realise that their prices are different from firms with sticky prices. To preserve relative prices, flexible-price firms reduce their prices to bring them in line with those in sticky-price sectors. As a consequence, inflation in the flexible-price sector depends negatively on relative price in that sector in the previous period. If relative price in that sector in the previous period is too high, flexible-price firms cut their prices significantly. This behaviour makes price-setting history dependent in the sense that inflation in the flexible-price sector and, in turn, aggregate inflation depend on lagged relative prices. Relative prices are related to output. Therefore, output in the previous period becomes relevant. This history dependence, in the presence of persistent monetary policy shocks, leads to a hump-shaped response in output.

The remainder of the chapter is organised as follows. Section 2 presents the model and describes our calibration of the model. Section 3 solves the model using the undetermined coefficient method. Section 4 reports impulse response functions (IRF) of output from the version of the model calibrated based on the Bils and Klenow (2004) dataset. Section 5 performs robustness checks. Finally, Section 6 concludes.

2.2 The Multiple Calvo (MC) model

The model is the simplified version of that in Kara (2015) and incorporates heterogeneity in price stickiness into an otherwise standard New Keynesian model, using the multiple Calvo approach. The economy consists of the following set of agents: firms, households, and a monetary authority. As in the standard model, there is a continuum of firms and households over the unit interval. Monopolistically competitive firms, indexed by $f \in [0, 1]$, produce differentiated goods and set prices according to Calvo (1983) pricing. Households, indexed by $h \in [0, 1]$, consume the final consumption good and supply labour. Each household h is twinned with each

firm f , which implies that the household h can only work for firm f . The unit interval of firms and households are divided into sectors, indexed by i ($= 1, \dots, N$). In sector i , the hazard rate is given by $1 - \delta_i$ and the share of each sector is given by α_i . A monetary authority sets nominal interest rate according to the Taylor rule. Below, we list log-linearised equations of the model economy. All variables are written as log-deviations from zero-inflation steady state. Let us start by describing inflation dynamics in sector i :

$$\pi_{it} = \beta E_t \pi_{it+1} + \kappa_i \gamma y_t - \kappa_i p_{it} \quad (2.1)$$

where $\kappa_i = (1 - \delta_i)(1 - \beta\delta_i)/\delta_i$. β is the subjective discount rate. γ shows the degree of real price rigidity in the economy. π_{it} denotes inflation in sector i and p_{it} the relative price in that sector. y_t denotes aggregate output. The demand for firms' output in sector i (y_{it}) is given by

$$y_{it} = -\epsilon p_{it} + y_t \quad (2.2)$$

where ϵ is the elasticity of substitution between different goods. The following equation relates sectoral inflation to aggregate inflation:

$$p_{it} = p_{it-1} + \pi_{it} - \pi_t \quad (2.3)$$

Since relative prices across sectors sum up to zero, we have

$$0 = \sum_{i=1}^N p_{it} \quad (2.4)$$

Aggregate output is given by the Euler equation

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) \quad (2.5)$$

where σ denotes the degree of relative risk aversion. Finally, the monetary authority follows the simple Taylor rule, which is given by

$$i_t = \phi_\pi \pi_t + \phi_y y_t + m_t \quad (2.6)$$

where m_t is a monetary policy shock and follows an AR(1) process.

$$m_t = \rho_m m_{t-1} + \epsilon_t \quad (2.7)$$

where ϵ_t is an iid shock with zero mean.

2.2.1 Choice of Parameters

We describe our calibration of the model. The time period corresponds to a quarter. The discount factor is set to $\beta = 0.99$ and the relative-risk-aversion parameter is set to $\sigma = 1$. The coefficient on inflation in the Taylor rule is $\phi_\pi = 1.5$ and that on output is $\phi_y = 0.125$. The autoregressive parameter of the monetary shock is set to $\rho_m = 0.9$. The elasticity of substitution between different goods is $\epsilon = 6$. The elasticity of labour supply is assumed to be 1. With these assumptions, the implied elasticity of the marginal costs to output is $\gamma = 0.3$. These are all standard values in the literature (see Ascari (2000) and Woodford (2003)). In Section 5, we report robustness checks that test the sensitivity of main conclusions to different parameter values. Finally, the frequency of price adjustment is calibrated based on the dataset provided by Bils and Klenow (2004). The mean age of contracts is 2.4 quarters; $\delta = 1 - 1/2.4$, which is used to calibrate the Calvo model.

2.3 A hump in output in the MC

In this section, using the undetermined coefficient method, we analytically show that the MC has a potential to generate a hump-shaped output response to monetary policy shocks. Before proceeding to our analysis, we make two simplifying

assumptions throughout Section 3. First, $\beta = 1$, $\phi_y = 0$ ¹. Second, we focus on a special MC with two sectors. In sector 1, prices are perfectly flexible, while they are sticky in sector 2. Given that prices adjust every period in sector 1, inflation in this sector does not have a Phillips curve representation. The relative price in this sector is given by

$$p_{1t} = \gamma y_t \quad (2.8)$$

Using this equation along with Equation (2.3), inflation in sector 1 can be expressed as

$$\pi_{1t} = \gamma(y_t - y_{t-1}) + \pi_t \quad (2.9)$$

Inflation in sector 2 is given by Equation (2.1)

$$\pi_{2t} = E_t \pi_{2t+1} + \kappa_2 \gamma y_t - \kappa_2 p_{2t} \quad (2.10)$$

Substituting Equation (2.9) into $\pi_t = \alpha_1 \pi_{1t} + \alpha_2 \pi_{2t}$ gives aggregate inflation in the MC.

$$\pi_t = \pi_{2t} + \frac{\alpha_1}{\alpha_2} \gamma (y_t - y_{t-1}) \quad (2.11)$$

As this equation clearly shows inflation is less forward-looking than that in the Calvo model. This equation gives inflation in the Calvo model when $\alpha_1 = 0$. When $\alpha_1 = 0$, the second term on the RHS disappears, resulting in a purely forward-looking equation. In the MC, inflation depends on the growth rate of output and, therefore lagged output. By combining Equations (2.4), (2.8), (2.10), and (2.11), we obtain the Phillips curve for this economy, which is given by

$$\pi_t = E_t \pi_{t+1} - \frac{\alpha_1}{\alpha_2} \gamma E_t y_{t+1} + \left\{ 2 \frac{\alpha_1}{\alpha_2} + \kappa_2 \left(1 + \frac{\alpha_1}{\alpha_2} \right) \right\} \gamma y_t - \frac{\alpha_1}{\alpha_2} \gamma y_{t-1} \quad (2.12)$$

Finally, combining this equation with the Euler equation (Equation (2.5)), the Taylor rule (Equation (2.6)), and the monetary shock process (Equation (2.7)), we

¹We numerically test our results with more realistic values for these parameters ($\beta = 0.99$, $\phi_y = 0.125$). Our results do not change significantly.

express output in terms of its own lag, leads, and the shock²

$$y_t = \tau_1 y_{t-1} + \tau_2 E_t y_{t+1} + \tau_3 E_t y_{t+2} + \tau_4 m_t \quad (2.13)$$

$$\begin{aligned} \text{where } \tau_1 &= \phi_\pi \frac{\alpha_1}{\alpha_2} \gamma / \bar{\tau}_1 \\ \tau_2 &= (2 + \phi_\pi \frac{\alpha_1}{\alpha_2} \gamma + \bar{\tau}_2) / \bar{\tau}_1 \\ \tau_3 &= -(1 + \frac{\alpha_1}{\alpha_2} \gamma) / \bar{\tau}_1 \\ \tau_4 &= -(1 - \rho_m) / \bar{\tau}_1 \\ \bar{\tau}_1 &= 1 + \phi_\pi \bar{\tau}_2 + \frac{\alpha_1}{\alpha_2} \gamma \\ \bar{\tau}_2 &= \{\kappa_2 + (2 + \kappa_2) \frac{\alpha_1}{\alpha_2}\} \gamma \end{aligned}$$

The coefficient on lagged output depends on the relative share of the flexible-price sector (α_1/α_2), the coefficient on inflation in the Taylor rule (ϕ_π), nominal rigidity (κ_2) and real rigidity (γ) parameters. When $\alpha_1 = 0$, $\tau_1 = 0$. This equation gives the output dynamics in the standard Calvo model. We solve the model by employing the undetermined coefficient method, which involves guessing the general functional form of the solution and then using the model to determine the coefficients. We guess that y_t is a linear function of y_{t-1} and m_t

$$y_t = \chi_y y_{t-1} + \chi_m m_t \quad (2.14)$$

The coefficients on Equation (2.14) are given by the following system of equations.

$$\tau_3 \chi_y^3 + \tau_2 \chi_y^2 - \chi_y + \tau_1 = 0 \quad (2.15)$$

$$\chi_m (\tau_3 \chi_y^2 + \tau_2 \chi_y + \tau_3 \rho_m \chi_y + \tau_2 \rho_m + \tau_3 \rho_m^2 - 1) + \tau_4 = 0 \quad (2.16)$$

Since the exact analytical results require to solve a cubic equation, we rely on numerical simulations to study the effects of different parameters on output. Our first result is that the model becomes more backward-looking, as the share of the flexible-price sector increases in the economy. In the first panel of Figure 2.1, we plot

²See Appendix for a detailed derivation.

the coefficient on lagged output (χ_y) against the flexible-price sector share (α_1)³. As it is evident from the figure, the coefficient on lagged output increases with the share of the flexible-price sector. The second panel indicates that the absolute value of the coefficient on the shock (χ_m) remains more or less the same when the flexible-price sector share is between 0.01 and 0.4, and then decreases with the sector share. Output responses become more persistent, as the flexible-price sector share increases, since χ_y increases. An increased flexible-price sector share reduces the effect of the shock on impact, since $|\chi_m|$ decreases.

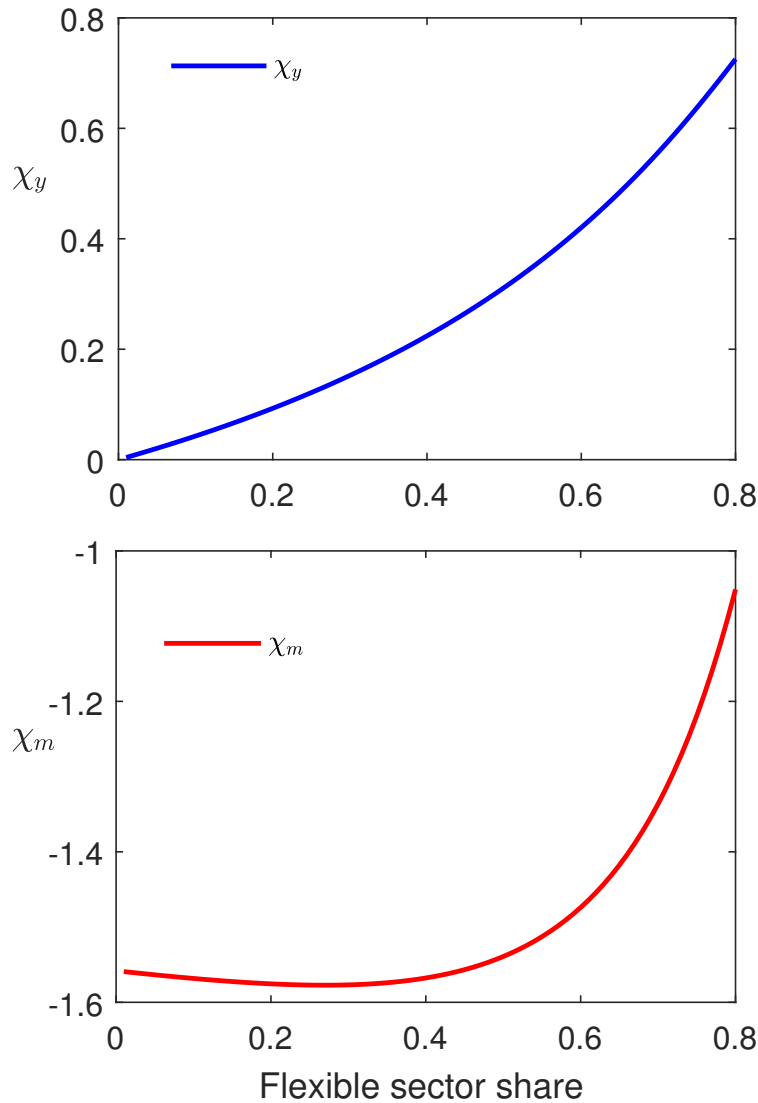


Figure 2.1: Flexible-price sector share and the coefficients on the output equation

Note: The first panel shows the coefficient on lagged output (χ_y) and the second panel shows the coefficient on the shock (χ_m).

³Note that the value of χ_y does not depend on the value of ρ_m .

Since output dynamics is determined by the coefficients χ_y and χ_m , we can derive a condition for a hump in output in terms of these two coefficients. A hump-shaped response requires that $y_{t+1} > y_t$. From Equation (2.14), in period t , output is given by $y_t = \chi_m m_t$. In period $t + 1$, $y_{t+1} = \chi_y y_t + \chi_m m_{t+1}$. Using Equation (2.7) and the fact that $\epsilon_{t+1} = 0$, we obtain $y_{t+1} = \chi_y y_t + \chi_m \rho_m m_t$. Given that $y_t = \chi_m m_t$ and $y_{t+1} = \chi_y \chi_m m_t + \chi_m \rho_m m_t$, straightforward algebra gives the following condition for a hump-shaped output response to monetary shocks:

$$\chi_y > 1 - \rho_m \quad (2.17)$$

This condition suggests that if the shock process is sufficiently persistent, the MC can generate a hump-shaped response. Figure 2.2 shows the minimum required value of the shock persistence when we vary the share of the flexible-price sector. As the figure shows, an increase in the flexible-price sector share reduces the need for highly persistent monetary policy shock. For example, when $\alpha_1 = 0.1$, the degree of persistence has to be 0.96. On the other hand, when the flexible-price sector share is 0.5, it is lower at 0.69.

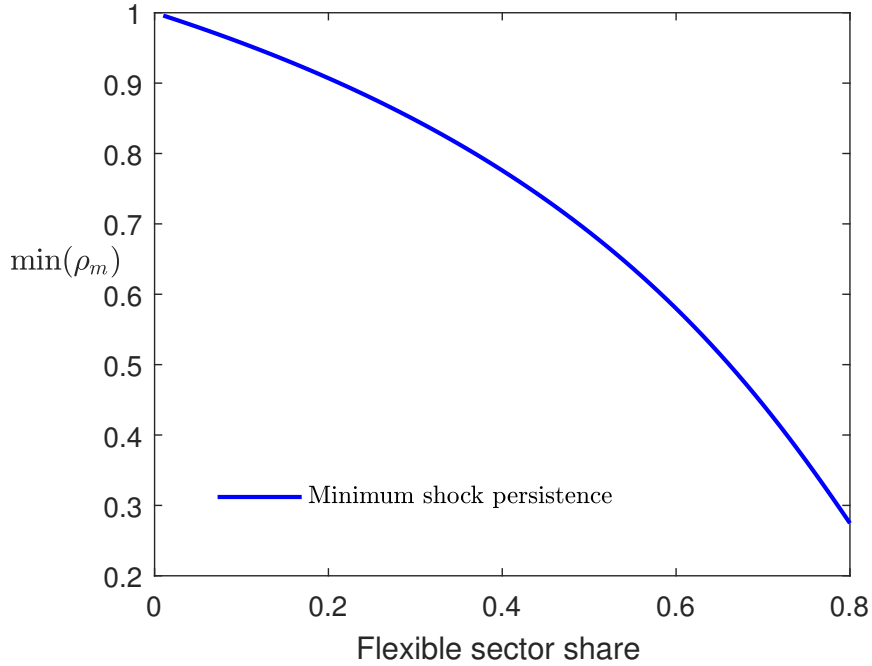


Figure 2.2: Flexible-price sector share and the minimum shock persistence

Note: The minimum shock persistence required for an output hump is calculated using the condition $\chi_y > 1 - \rho_m$, for each value of the flexible-price sector share.

Finally, when the flexible-price sector share is 0, the model becomes the Calvo model. In this case, Equation (2.13) has no lagged output in it, and there is no hump for any value of ρ_m . For $0 < \rho_m < 1$, output increases on impact and then monotonically goes back to the initial steady-state level. For $\rho_m > 1$ output increases infinitely, leading to indeterminacy.

2.4 Impulse response functions of output

This section reports the IRFs of output in response to the monetary shock. We report the IRFs from a 2-sector MC and from a more realistic version of the MC, which uses the distribution of contract lengths provided by Bils and Klenow (2004) (BK-MC). We begin with the 2-sector MC. Figure 2.3 shows the IRF of output to a monetary policy shock from the MC for different shares of the flexible-price sector ($\alpha_1 = 0.3$, $\alpha_1 = 0.6$, and $\alpha_1 = 0.8$), along with that from the Calvo model. The results are consistent with those reported in the previous section. Two points should be made. First, a hump in output becomes bigger as the share of the flexible-price sector increases. Second, the initial response of output becomes more muted, as the share increases. These two results reflect the fact that the coefficient on lagged output (χ_y) becomes greater and that on the shock (χ_m) becomes smaller, as the flexible-price sector share increases.

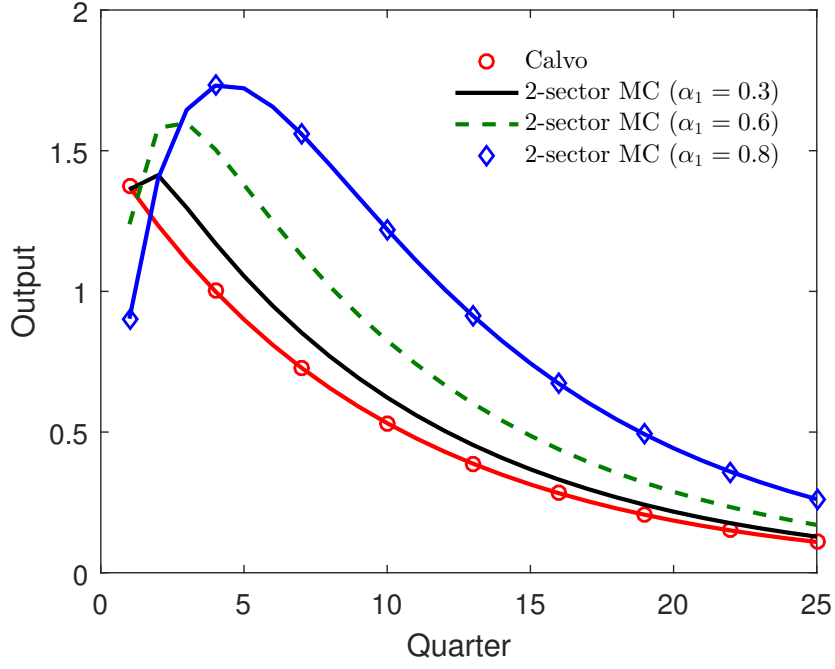


Figure 2.3: Impulse Response Functions to monetary policy shock in 2-sector MC model

Note: The solid red line with circles denotes the IRF of output from the Calvo model. The solid black line and the dashed green line denote the IRF from the 2-sector MC model when the flexible-price sector share is 0.3 and 0.6, respectively. The solid blue line with diamonds denotes the IRF from the 2-sector MC model when the flexible-price sector share is 0.8.

Next, in Figure 2.4, we report the IRF of output from the BK-MC model.

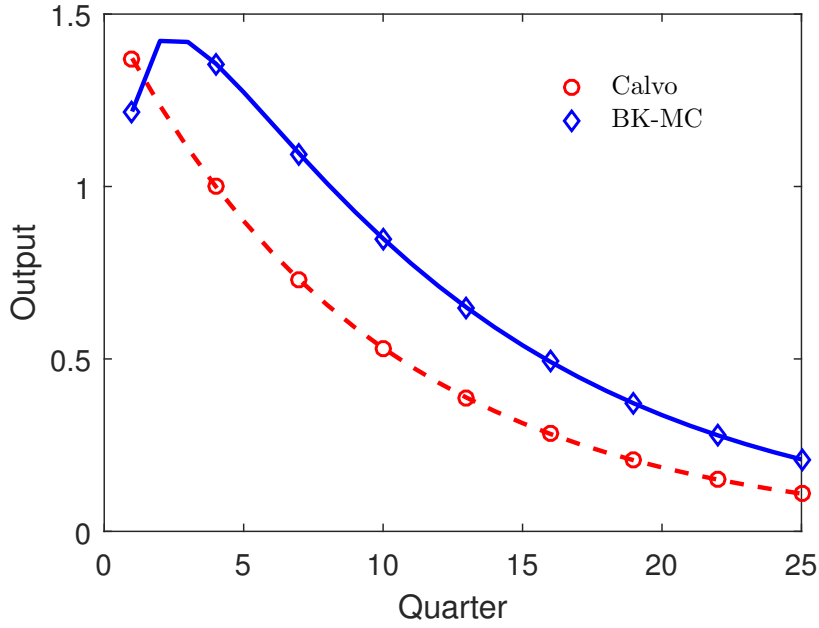


Figure 2.4: Impulse Response Functions to monetary policy shock in the BK-MC model

Note: The dashed red line with circles denotes the IRF of output from the Calvo model. The solid blue line with diamonds denotes the IRF from the BK-MC model.

The figure also plots the IRF from the Calvo model. In the Calvo model, the maximum effect of the shock is on impact and there is no hump. In contrast, the BK-MC generates a hump-shaped response. These results suggest that our results for the simplified MC hold true in the version of the model with a more realistic distribution.

2.5 The role of different parameter values

Output dynamics in the MC depends on two key parameter values; the degree of real rigidity (γ) and the coefficient on inflation in the Taylor rule (ϕ_π), as is seen in Equation (2.13). Therefore, we check the robustness of our conclusions for plausible range of values of the two parameters. We perform these tests using a 2-sector MC. We consider different calibrations and for each calibration, given the flexible-price sector share, we calculate the minimum degree of monetary shock persistence needed by the model to generate a hump-shaped output response. Figure 2.5 reports the results from this experiment. The first panel reports four calibrations where we vary the degree of real rigidities γ between 0.1 and 0.4. When γ is lower (e.g. $\gamma = 0.2$) the MC requires more persistent shock to generate a hump. The second panel also reports four calibrations where we vary the coefficient on inflation in the Taylor rule (ϕ_π) between 1.2 and 2.5. A stronger reaction by the central bank to inflation (e.g., $\phi_\pi = 2.5$) reduces the required degree of monetary shock persistence. However, in all cases we consider in this section, the results are very similar to those in the benchmark case.

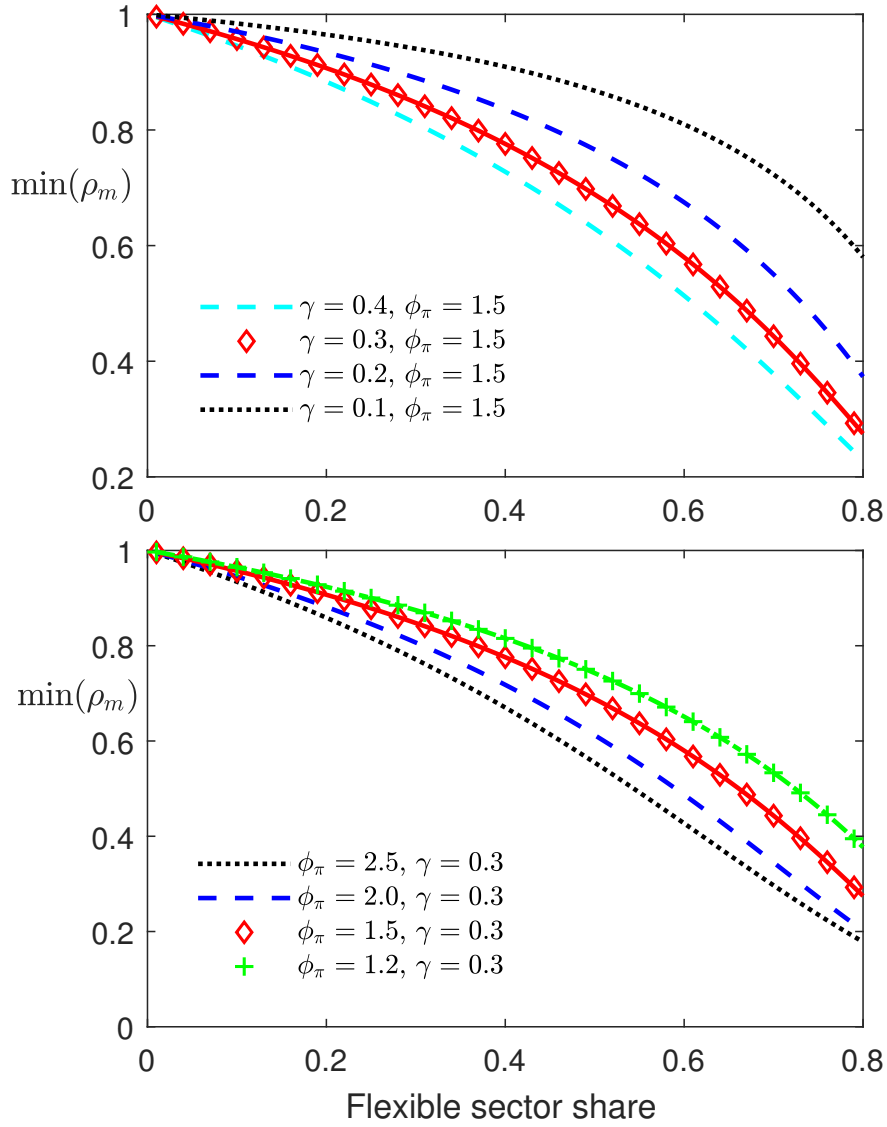


Figure 2.5: Flexible-price sector share and the minimum shock persistence, with different calibrations

Note: The first panel reports the required degree of monetary shock persistence in the 2-sector MC for different values of γ , which represents the degree of real rigidities. The second panel shows the required degree of shock persistence for different values of ϕ_π , which denotes the coefficient on inflation in the Taylor rule. How to calculate the minimum shock persistence is described in the notes of Figure 2.2.

2.6 Summary and Conclusions

We have proposed a solution to a long-standing challenge in macroeconomics – namely that output follows a hump-shaped pattern in response to monetary policy shocks. We have shown that a New Keynesian model that accounts for heterogeneity

in price stickiness can generate a hump-shaped response in output when monetary policy shocks are persistent. The presence of fully-flexible prices is the key for this result. Such prices are more responsive to shocks than sticky prices. Prices in the flexible-price sector are high, relative to those in sticky sectors when the shock hits the economy. To bring their prices in line with those in sticky sectors, firms in the flexible-price sector cut their prices soon after the shock. As result, inflation in the flexible-price sector and, therefore, aggregate inflation depend on lagged relative prices in a multi-sector model. This introduces inertia to price-setting. This inertia, along with persistent monetary policy shocks, leads to a hump-shaped response in output to monetary policy shocks.

Chapter 3

Output dynamics in New Keynesian models with search frictions and heterogeneity in nominal rigidities

3.1 Introduction

Many empirical studies, e.g. Sims (1986), Bernanke and Blinder (1992), and Christiano, Eichenbaum, and Evans (1996, 1999), estimate the quantitative effects of a shock to monetary policy by using typically vector autoregressions. The consensus from these studies is that a monetary policy shock leads to a hump-shaped response in aggregate real quantities, including output. The peak effect of a shock on output occurs not on impact of the shock, but several quarters after the shock. As Christiano, Eichenbaum, and Trabandt (2017) state, a common prescription to this problem is to assume habit-formation in consumption. For example, widely-used medium-sized New Keynesian business-cycle models such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) adopt this assumption. Habit-formation generates inertia in the model that is required for hump-shaped responses in real activities.

This chapter provides an alternative approach to the problem. We extend a simple New Keynesian model with both price and wage stickiness by adding (i) the search and matching frictions in the labour market as in Diamond (1982a,b), Mortensen (1982), and Pissarides (1985)¹ and (ii) heterogeneity in nominal rigidi-

¹See also Mortensen and Pissarides (1994).

ties. In this model, the interactions between our two extensions generate inertia in output dynamics, which leads to a hump-shaped output response when a monetary shock is persistent. In particular, we show that long-term employment relationship that arises under the search and matching framework gives rise to the history dependence in marginal costs: marginal costs depend on lagged output level. In conjunction with heterogeneity in nominal rigidities, the search frictions produce rich inflation and output dynamics in the model. By the method of undetermined coefficients, it is revealed that output shows an inertial response in our model. That is, output can be expressed as an autoregressive process, where it depends on its own lags and a monetary shock process. We find that the coefficient on lagged output increases with the degree of long-term employment relationship. Further, the coefficient on lagged output increases faster with this long-term relationship, when heterogeneity in nominal rigidity is stronger, and *vice versa*. This suggests that the search frictions and heterogeneity in nominal rigidities reinforce each other to help the model reproduce inertial output dynamics.

The key to our results is the role of long-term employment relationship, that arises from the search and matching framework, for inflation and output dynamics. Search and matching models explicitly stipulate that previously-employed workers stay in the job for a long period of time. In a model with the standard labour market, where hiring is costless, this would not matter. But with a costly and time-consuming hiring process, the (partial) continuation of previously created job matches implies that firms could save hiring costs in the current period thanks to past hiring activities, and hence, with other things equal, marginal costs would be lower if they hired more workers in the past. As a result, lagged employment, and lagged output level affect marginal costs in the current period. To further illustrate this point, suppose that there is a hike in hiring activity after an expansionary shock. In the next period, many of the increased stock of workers will stay in the job unless separated, and therefore the demand for new hires decreases substantially. This means hiring costs, that constitute marginal costs together with wages, increase steeply after an expansionary monetary shock, and then fall considerably. Inflation

depends on marginal costs, and on lagged output level. Therefore previous path of output becomes relevant. This history dependence results in inertia in the model, leading to a hump-shaped output response when monetary shock is persistent.

This chapter is closely related to our second chapter, where we extend a simple New Keynesian DSGE model with the standard labour market by introducing heterogeneity in price stickiness. In a model with multiple sectors, each with a different degree of price stickiness, we show that the presence of the flexible-price sector makes inflation history dependent and helps an otherwise standard model replicate hump-shaped output responses to persistent monetary shocks. This chapter shares the same goal, but does so in a richer environment. The departure is to introduce two realistic assumptions into the model: nominal wage stickiness and the search and matching frictions in the labour market. There is ample evidence that wages are sticky, and it is common to assume wage stickiness in medium-sized New Keynesian models as a device to reproduce large and persistent responses in real activities to monetary shocks. In addition, we revisit the implications of the search and matching framework in the labour market, which is a standard tool in current macro-labour analysis, within the New Keynesian environment. It is revealing that the model with heterogeneity in nominal rigidities and with the standard labour market cannot generate a hump-shaped output response to monetary shocks when wages are sticky, as shown in Section 4. We interpret this as suggesting the relevance of the search frictions in the New Keynesian models.

This chapter is related to the literature that studies the role of the search frictions in New Keynesian business cycle models. Walsh (2005) studies how the search and matching frictions affect the real effects of monetary policy shocks. He finds that the search frictions increase output responses relative to an otherwise similar model with the standard labour market. Trigari (2009) reports similar findings. She argues that the search frictions lower the elasticity of marginal costs with respect to output, and this helps the model explain the persistent output response to monetary shocks. In contrast, Krause and Lubik (2007) point out that the precise role of the search frictions in Walsh (2005) and Trigari (2009) are difficult to assess, due

to model assumptions other than search frictions such as habit formation. Based on an analysis of a simple New Keynesian model with the search frictions, they conclude that the search frictions *per se* do not affect the mechanism that generates endogenous persistence in the model.

Another study that is closely related to ours is Ravenna and Walsh (2008). They study the role of search frictions in the labour market for inflation. It is shown that the lagged unemployment term appears in the Phillips curve due to the search frictions. They also note that long-term employment relationship gives rise to the backward-lookingness in inflation. We go one step further and show the link between this backward-lookingness in inflation and output inertia, in the models with the search frictions, with and without heterogeneity in nominal rigidities.

The rest of the chapter is organised as follows. Section 2 presents the model. Section 3 provides our calibration. Section 4 presents the impulse response function (IRF) of output to monetary shocks in our models. Section 5 uses the method of undetermined coefficients to analytically show the role of the search frictions and its interaction with heterogeneity in nominal rigidity, focusing on inertial output responses. Section 6 concludes.

3.2 The model

3.2.1 Overview of the Economy

Our model builds on the RBC model with the search frictions in the labour market and heterogeneity in wage stickiness, that we developed in the first chapter. We bring the model into New Keynesian environment, and introduce heterogeneity in price stickiness. Therefore our model features the search and matching frictions in the labour market and heterogeneity in price and wage stickiness. The economy consists of the following set of agents: retailers, firms, households, a monetary authority, and the government. Note that we distinguish two types of firms: retailers and wholesale firms, or simply firms. We do this to separate the staggered price setting from the search and matching frictions and wage bargaining. This setup is widely used in New

Keynesian models with the search frictions, e.g., Trigari (2009) and Walsh (2005) as well as models with the standard labour market, e.g. Smets and Wouters (2007), where the firms in our model correspond to the craft-unions in those models.

There is a continuum of firms and retailers over the unit interval. Monopolistically competitive firms, indexed by $f \in [0, 1]$, hire workers, subject to the search frictions, to produce differentiated wholesale goods which are sold to retailers. Firms set prices flexibly and Nash-bargain their wage rates with workers subject to Calvo (1983) staggering. Monopolistically competitive retailers, indexed by $j \in [0, 1]$, sell differentiated consumption goods to households, subject to price rigidity. The households consume final consumption goods and supply labour to firms. Each household has a continuum of members, who can either be workers or unemployed. Firms are divided into sectors with index i ($= 1, \dots, N$) and with the shares α_i . Retailers are also divided into sectors with index k ($= 1, \dots, M$) and with the shares α_k . The sectors are differentiated by their degree of wage stickiness in case of firms, and that of price stickiness in case of retailers. Without loss of generality, we assume that the degree of nominal rigidities increase with indices i and k . It is assumed that workers can freely move from one sector to another to find a job when they are unemployed. A monetary authority sets nominal interest rate according to a policy rule, subject to shocks. The government pays unemployment benefit to households by raising lump-sum taxes.

3.2.2 The labour market and the matching

The labour market structures and the search and matching process are standard², and the search and matching is done at the economy-wide level. Let n_{fit-1} be the number of workers at firm f in sector i in period $t-1$. Each firm loses an exogenously given fraction λ of workers in each period. At the same time, the firm posts vacancies v_{fit} to hire new workers. The aggregate vacancies (v_t) and the unemployed workers (u_t) meet to produce successful matches³. The matching is given by the aggregate matching function $m(u_t, v_t) \equiv \mu_m u_t^\mu v_t^{1-\mu}$, where μ_m is the scaling parameter. Labour

²See Pissarides (2000) for a textbook presentation of the model.

³ $v_t \equiv \int_0^1 v_{fit} df$ and $u_t = 1 - n_{t-1}$.

market tightness is defined as $\theta_t \equiv v_t/u_t$. We also define $q(\theta_t)$ as the vacancy-filling rate and $p(\theta_t)$ as the job-finding rate. The vacancy posting comes at a flow cost κ for the duration of the vacancy. We assume that new hires participate in production immediately⁴.

3.2.3 Households

The household is characterized by the standard constant-relative-risk-aversion preference and a budget constraint. We follow Merz (1995) and assume that income is pooled together within the representative household. Then the household's optimisation problem yields the Euler equation.

$$1 = \beta E_t \left[(1 + i_t) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \quad (3.1)$$

where σ denotes the constant-relative-risk-aversion parameter and β the subjective discount rate. i_t is the nominal interest rate between period t and $t + 1$. The final consumption good is a Dixit-Stiglitz aggregate of differentiated retail goods; $c_t = \left(\int_0^1 c_{jkt}^{(\epsilon^p - 1)/\epsilon^p} dj \right)^{\epsilon^p/(\epsilon^p - 1)}$, where ϵ^p denotes the elasticity of substitution between differentiated retail goods. The corresponding price index (P_t) is given by $P_t = \left(\int_0^1 P_{jkt}^{1 - \epsilon^p} dj \right)^{1/(1 - \epsilon^p)}$, where P_{jkt} is the retailer j 's price. Then the demand for the retailer j 's variety is given by

$$c_{jkt} = \left(\frac{P_{jkt}}{P_t} \right)^{-\epsilon^p} c_t \quad (3.2)$$

Additionally, we define $J_{fit}^W(w_{fit})$ as the workers' surplus from a job at firm f , i.e., the marginal value of an additional employment to the household.

$$J_{fit}^W(W_{fit}) = \frac{W_{fit}}{P_t} - b - E_t \beta_{t,t+1} [p(\theta_{t+1}) J_{xt+1}^W - (1 - \lambda) J_{fit+1}^W(W_{fit+1})] \quad (3.3)$$

where $J_{xt+1}^W \equiv \int_0^1 \frac{v_{qit+1}}{v_{t+1}} J_{qit+1}^W(W_{qit+1}) dq$ denotes the average surplus of a worker who is newly hired in time $t + 1$. $\beta_{t,t+s} \equiv \beta c_{t+s}^{-\sigma}/c_t^{-\sigma}$ is the stochastic discount factor between periods t and $t + s$.

⁴The same assumption is made in Blanchard and Gali (2010) and Gertler, Sala, and Trigari (2008).

3.2.4 Firms

Firms hire workers to produce differentiated wholesale goods. Let us denote the output of the firm f in sector i by l_{fit} . We assume the following constant-return-to-scale production technology.

$$l_{fit} = n_{fit} \quad (3.4)$$

Aggregate wholesale good l_t is a Dixit-Stiglitz aggregate of differentiated wholesale goods; $l_t = \left(\int_0^1 l_{fit}^{(\epsilon^w-1)/\epsilon^w} df \right)^{\epsilon^w/(\epsilon^w-1)}$, where ϵ^w denotes the elasticity of substitution between differentiated wholesale goods. The corresponding wholesale price index (P_t^w) is given by $P_t^w = \left(\int_0^1 P_{fit}^{1-\epsilon^w} df \right)^{1/(1-\epsilon^w)}$, where P_{fit} is the firm f 's price. Then the demand for firm f 's variety is

$$l_{fit} = \left(\frac{P_{fit}}{P_t^w} \right)^{-\epsilon^w} l_t \quad (3.5)$$

As explained above, the workforce of firm f in sector i consists of workers that have been employed in the past and new hires.

$$n_{fit} = (1 - \lambda)n_{fit-1} + q(\theta_t)v_{fit} \quad (3.6)$$

In each period, the firm maximises the following firm value subject to Equations (3.4), (3.5), and (3.6), by choosing price (P_{fit}), employment (n_{fit}), and vacancy (v_{fit}), taking as given the employment at the beginning of the period (n_{fit-1}), the labour market tightness (θ_t) and the current and expected path of wages (W_{fit}).

$$F_{fit} \equiv \max_{P_{fit}, n_{fit}, v_{fit}} E_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left[\frac{P_{fit+s}}{P_{t+s}} l_{fit+s} - \frac{W_{fit+s}}{P_{t+s}} n_{fit+s} - \kappa v_{fit+s} \right] \quad (3.7)$$

The solution to this problem yields the job creation condition.

$$\frac{\kappa}{q(\theta_t)} = \varphi_{fit} - \frac{W_{fit}}{P_t} + (1 - \lambda)E_t \beta_{t,t+1} \frac{\kappa}{q(\theta_{t+1})} \quad (3.8)$$

φ_{fit} is the Lagrange multiplier on the output constraint. This implies the contribution of an additional unit of output to firm value, and therefore equals to firm's real

marginal costs. The firm's pricing decision is given by

$$p_{fit} \equiv \frac{P_{fit}}{P_t} = \frac{\epsilon^w}{\epsilon^w - 1} \varphi_{fit} \quad (3.9)$$

That is, the firm adds a constant mark-up over marginal costs to set prices.

We define $J_{fit}^F(W_{fit})$ as the firm's marginal surplus from a job when it hires an additional worker at the wage rate W_{fit} . This is given by

$$J_{fit}^F(W_{fit}) = \varphi_{fit} - \frac{W_{fit}}{P_t} + (1 - \lambda) E_t \beta_{t,t+1} J_{fit+1}^F(W_{fit+1}) \quad (3.10)$$

This implies that the firm's surplus is equal to the sum of the net marginal revenue in the current period and the continuation value of the match.

3.2.5 Wage bargaining

Wage rate is Nash bargained. Then the sharing rule is given by

$$J_{fit}^W(W_{fit}) = \eta [J_{fit}^W(W_{fit}) + J_{fit}^F(W_{fit})] \quad (3.11)$$

where η denotes the worker's bargaining power⁵. In case of period-by-period wage bargaining as in standard search and matching models, we obtain real wages (w_t) by substituting Equations (3.3) and (3.10) into (3.11)⁶.

$$w_t = w_t^{FLEX} \quad (3.12)$$

where

$$w_t^{FLEX} = \eta \varphi_t + (1 - \eta) \{b + E_t \beta_{t,t+1} p(\theta_{t+1}) J_{xt+1}^W\} \quad (3.13)$$

We denote flex wages, i.e., the solution to the Nash bargaining problem when wages are fully flexible, by w_t^{FLEX} . As in standard search and matching models, Equation (3.13) implies that flex wage is a weighted average of the workers' contri-

⁵As in Thomas (2008), we exclude the horizon effect, which results from the fact that workers and firms have different horizons when negotiating wages. Gertler and Trigari (2009) report that this effect is not significant.

⁶We drop the firm and the sector subscripts, since all firms set the same wage rates when all wages are bargained each period.

bution to the match and the workers' opportunity costs.

In case of staggered wage bargaining, only a fraction $1 - \delta_i^w$ of firms in sector i renegotiate nominal wages, denoted by W_{it}^* ⁷, with workers in each period. We assume that newly hired workers receive the same wage with the continuing workers if they enter the firm between the contracts⁸. Therefore, when negotiating their wages, workers and firms consider the fact that the wage is going to remain effective for some time. We adjust the firm and the worker surpluses (Equation (3.10) and (3.3)) to reflect this fact by applying $E_t J_{it+1} = E_t [\delta_i^w J_{it+1}(W_{it}^*) + (1 - \delta_i^w) J_{it+1}(W_{it+1}^*)]$ for both worker and firms surpluses, and substitute the resulting expressions into Equation (3.11). This yields an expression for real reset wages w_{it}^* in sector i .

$$w_{it}^* = \sum_{s=0}^{\infty} E_t \psi_{i,s} w_{it+s}^{FLEX} \quad (3.14)$$

where

$$\psi_{i,s} \equiv \frac{\{\delta_i^w(1 - \lambda)\beta\}^s \Lambda_{t,t+s}}{\sum_{q=0}^{\infty} \{\delta_i^w(1 - \lambda)\beta\}^q \Lambda_{t,t+q} / \Pi_{t,t+q}} \quad (3.15)$$

and

$$\Lambda_{t,t+s} \equiv c_{t+s}^{-\sigma} / c_t^{-\sigma}, \quad \Pi_{t,t+q} \equiv \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \dots \frac{P_{t+q}}{P_{t+q-1}} \quad (3.16)$$

This equation implies that real reset wage is a weighted average of the current and future flex wages over the expected life of contracts. Since firms consider the cases when their wage rates remain effective in the future, they consider not only current flex wages, but also future flex wages, discounted by the survival probability of wage (δ_i^w), along with the survival probability of job ($1 - \lambda$) and the subjective discount factor (β). They also consider the cumulative inflation ($\Pi_{t,t+s}$), because when nominal reset wages are once set, the real wage rate at future dates will fall due to inflation.

Lastly, average nominal wage in sector i , denoted by W_{it} , is given by

$$W_{it} = \delta_i^w W_{it-1} + (1 - \delta_i^w) W_{it}^* \quad (3.17)$$

⁷Since all renegotiating firms in sector i set the same wage, we drop the subscript f .

⁸Gertler, Huckfeldt, and Trigari (2016) and Stuber (2017) provide empirical evidence that supports this assumption.

This equation implies that average wage in sector i is a weighted average of reset wages and past wages, with the weight given by the reset probability.

3.2.6 Retailers and price setting

Following Smets and Wouters (2007), we assume that all retailers use the same mix of wholesale goods. Retailers repackage the aggregate wholesale goods to produce differentiated consumption goods, and monopolistically set prices. For simplicity, we assume that retailers have no other inputs or costs and their output is given by following constant-return-to-scale technology.

$$y_{jkt} = l_{jkt} \quad (3.18)$$

where y_{jkt} denotes the output of retailer j and l_{jkt} the retailer's demand for the aggregate wholesale goods. In each period, a fraction $1 - \delta_k^p$ of retailers reoptimise their prices. Then the real reset price in sector k ⁹ ($\frac{P_{kt}^*}{P_t}$) is

$$\frac{P_{kt}^*}{P_t} = \frac{\epsilon^p}{\epsilon^p - 1} \sum_{s=0}^{\infty} E_t \xi_{i,s} m c_{t+s} \quad (3.19)$$

where

$$\xi_{i,s} \equiv \frac{(\delta_k^p \beta)^s c_{t+s}^{1-\sigma} \Pi_{t,t+s}^{\epsilon^p}}{\sum_{q=0}^{\infty} (\delta_k^p \beta)^q c_{t+q}^{1-\sigma} \Pi_{t,t+q}^{\epsilon^p - 1}} \quad (3.20)$$

Note that the real marginal cost of retailers is given by the real price of the aggregate wholesale goods.

$$m c_t = \frac{P_t^w}{P_t} \quad (3.21)$$

The average price in sector k (P_{kt}) is given by

$$P_{kt}^{1-\epsilon^p} = \delta_k^p P_{kt-1}^{1-\epsilon^p} + (1 - \delta_k^p) P_{kt}^{*1-\epsilon^p} \quad (3.22)$$

This implies that (log-linearised) sectoral average price is a weighted average of reset prices and past prices, with the weight given by the reset probability.

⁹We drop the retailer subscript j , since all reoptimising retailers set the same price.

3.2.7 The rest of the model

The monetary policy is characterized by a simple Taylor rule.

$$I_t = (P_t/P_{t-1})^{\phi_\pi} (y_t/\tilde{y})^{\phi_y} m_t \quad (3.23)$$

where I_t is the gross nominal interest rate and m_t is an exogenous monetary policy shock. The log of monetary policy shock follows an AR(1) process; $\ln m_t = \rho_m \ln m_{t-1} + \epsilon_t^m$, where $\rho_m \in [0, 1)$ and ϵ_t^m is an iid shock with zero mean. All output is consumed in equilibrium:

$$y_t = c_t \quad (3.24)$$

The government finances unemployment benefit (b) with taxes (T).

$$b(1 - n_t) = T_t \quad (3.25)$$

where $n_t \equiv \int_0^1 n_{fit} df$ is the aggregate employment.

3.3 Calibration

We choose parameter values that are standard in the business cycle models with the search frictions and in the estimated New Keynesian models. The time period corresponds to a quarter. The discount factor is set to $\beta = 0.99$ and the constant-relative-risk-aversion parameter is assumed to be $\sigma = 1$. We set the coefficient on inflation in the Taylor rule to $\phi_\pi = 1.5$ and that on output to $\phi_y = 0.125$. The autoregressive parameter of the monetary shock is set to $\rho_m = 0.9$. The elasticity of substitution between differentiated (retail and wholesale) goods is $\epsilon^p = \epsilon^w = 10$, which implies the steady-state markups of 11% for each firm and retailer.

Turning to the parameter values that are specific to the search and matching model, the separation rate is calibrated at $\lambda = 0.1$. This value is consistent with the evidence provided by Davis, Haltiwanger, and Schuh (1996) for the separation rate in the US. Similar values are adopted in Walsh (2005) and Krause and Lubik (2007). For the matching function, we set the match elasticity to $\mu = 0.5$. Workers' bargaining power is set to $\eta = 0.5$. We follow Walsh (2005) and set the job-finding

rate of workers to $p(\tilde{\theta}) = 0.6$. With the workforce size normalised to one, this assumption implies the steady-state number of unemployed job-seekers to be 0.15. This is to allow for the potential job seekers such as discouraged workers, since our model does not include the labour-market participation decision. Our choice of the number of job-seekers is in the middle of those in related studies: 0.12 in Krause and Lubik (2007) and 0.33 in Trigari (2009). The firm's vacancy-filling rate is set to $q(\tilde{\theta}) = 0.7$, following Krause and Lubik (2007) and Den Haan, Ramey, and Watson (2000). The unemployment benefit ratio (\tilde{b}) is the ratio of the unemployment flow value (b) to the worker's flow contribution to the match in the steady state ($\tilde{\varphi}$). Following Hall (2008), we set this parameter to $\tilde{b} = 0.7$. The flow cost of vacancy (κ) and unemployment benefit (b) are then implied by the job creation condition and the steady-state wage equation.

Finally, we turn to the frequency of price and wage adjustments. The frequency of price adjustment is calibrated based on the dataset provided by Bils and Klenow (2004). The dataset is based on US Consumer Price Index (CPI) microdata compiled by the Bureau of Labor Statistics. They provide the average proportion of price adjustments per period for each of the 350 product categories and the corresponding weights in the CPI. We interpret these proportions of adjustments as Calvo hazard rates. As in Kara (2015), we aggregate the 350 product categories into 10 sectors, each with a different hazard rate ($1 - \delta_k^p$), for computational ease¹⁰. The mean age of contracts is 2.4 quarters, which is used to calibrate the price-setting of the model without heterogeneity in price stickiness. Next, the frequency of wage adjustments is calibrated as follows. We calibrate the distribution of wage contract durations in our model using data compiled by Barattieri, Basu, and Gottschalk (2014). They provide evidence for the distribution of wage spells for the US economy. The data are based on the Survey of Income and Program Participation conducted by the US Census Bureau and are for the period from 1996 to 2000. As we did for the frequency of price adjustments, we assume the probability of wage change is constant, as in the Calvo model and interpret each wage spell reported in Barattieri, Basu, and Gottschalk (2014) as a Calvo wage reset probability in a given sector. In other

¹⁰See Kara (2015) for more details.

Table 3.1: Calibration

Subjective discount factor	β	0.99
Relative risk aversion	σ	1
Elasticity of substitution between goods	ϵ^p, ϵ^w	10
Coefficient on inflation in Taylor rule	ϕ_π	1.5
Coefficient on output in Taylor rule	ϕ_y	0.125
Monetary shock autoregressive parameter	ρ_m	0.9
Separation rate	λ	0.1
Vacancy-filling rate	$q(\tilde{\theta})$	0.7
Job-finding rate	$p(\tilde{\theta})$	0.6
Number of unemployed job-seekers	\tilde{u}	0.15
Elasticity of matching to unemployment	μ	0.5
Worker's bargaining power	η	0.5
Unemployment flow value	\tilde{b}	0.7
Mean hazard rate in price adjustment	$1 - \delta^p$	0.4
Mean hazard rate in wage adjustment	$1 - \delta^w$	0.18

Note: The number of unemployed job-seekers (\tilde{u}) are implied. The other parameters are fix.

words, we set $1 - \delta_i^w = 1/A_i$, where A_i is the age of sector i . The mean hazard rate is 0.18. We use this value to calibrate the models without heterogeneity in wage stickiness.

3.4 Impulse response functions

In this section, we study, in turn, the role of the search frictions and that of heterogeneity in nominal rigidities. Figure 3.1 shows the IRF¹¹ of output to monetary shocks in the models with and without the search frictions. Both models feature heterogeneity in nominal rigidities and wage rigidity. In the model with the stan-

¹¹Since our primary interest is the shape of output response, we normalise the responses in a way that the impact is set at one throughout this chapter, for a comparison purpose.

standard labour market, output increases on impact and then monotonically reverts to steady state. In contrast, in the models with the search frictions, output peaks 2 quarters after the shock. Apparently, the search frictions in the labour market have a potential to reproduce a hump-shaped output response to monetary shocks.

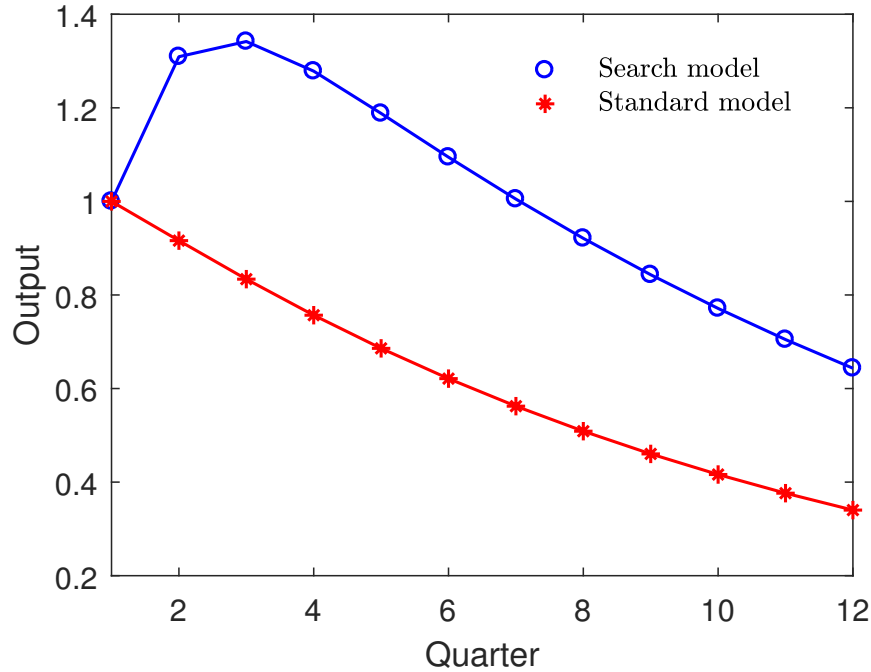


Figure 3.1: Impulse Response Functions to monetary policy shock in models with and without the search frictions

Note: The figure reports the output IRF to monetary shock from models with and without search frictions in the labour market. Both models include heterogeneity in price and wage stickiness. The IRFs are normalised so that the impact is set at one.

Next, we study the role of heterogeneity in nominal rigidities in models with the search frictions and both price and wage stickiness. Figure 3.2 shows the IRF of output to monetary shocks in five models: a model without heterogeneity in price stickiness and wage stickiness ('Calvo-Calvo search'), a model with heterogeneity in price stickiness, but without that in wage stickiness ('MC-Calvo search'), a model without heterogeneity in price stickiness but with that in wage stickiness ('Calvo-MC search'), and a model with heterogeneity in both price and wage stickiness ('MC-MC search'). For comparison we include previous result: a model with the standard labour market, with heterogeneity in both price and wage stickiness ('MC-MC standard'). Two points should be made. First, all models with the search frictions reproduce an output hump, while the MC-MC standard model does not.

Importantly, the Calvo-Calvo search model shows a hump in output even though the model does not include heterogeneity in nominal rigidities. This suggests that the search frictions *alone* has potential to help the model generate a hump in output. Second, heterogeneity in nominal rigidities significantly reinforces this potential. Output peaks 1 quarter after the shock in the Calvo-Calvo search model, while it does 2 quarters after the shock in the MC-MC search model. Further, the peak output response is about 35% higher than that on impact in the MC-MC search model, while it is only about 15% higher in the Calvo-Calvo search model. The models with heterogeneity in either price (MC-Calvo search model) or wage stickiness (Calvo-MC search model) fare better than the Calvo-Calvo search model, but are not as good as the MC-MC search model. This suggests that heterogeneity in both price and wage stickiness is important for a hump-shaped output dynamics.

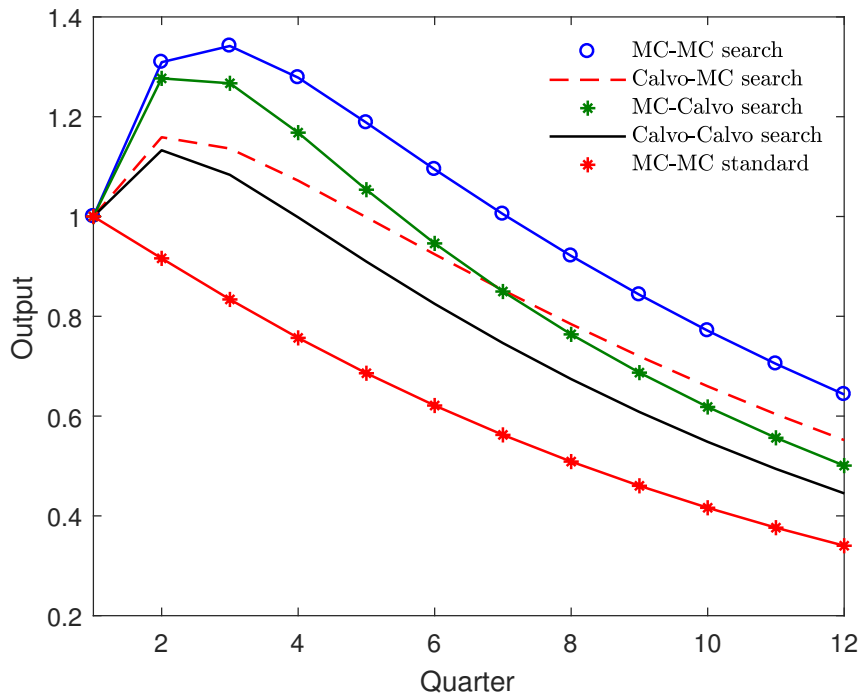


Figure 3.2: Impulse Response Functions to monetary policy shock in models with the search frictions

Note: The figure reports the output IRF to monetary shock from models with the search frictions in labour market. The models are different in whether there is heterogeneity in price and/or wage stickiness. The IRFs are normalised so that the impact is set at one.

3.5 Discussion

In this section, we explain how the search frictions make the model generate a hump-shaped output response to monetary shocks. First, we show that the search frictions make retailers' marginal costs history dependent. In particular, it is shown that the long-term employment relationship that arises from the search and matching frictions leads to this. Second, we show that the interaction between long-term employment relationship and heterogeneity in nominal rigidity results in rich output dynamics in the model. The method of undetermined coefficients reveals that inertial output response to monetary shocks arises from this interaction.

To clearly illustrate the role of hiring frictions and its interaction with heterogeneity in price stickiness, we make three simplifying assumptions. First, we assume that real wages are fixed: $W_t = \tilde{W}$ and $\hat{w}_t = 0$, where variables with a hat denote the log-deviations from the steady-state values. Second, we assume $\beta = 1$, $\sigma = 1$, and $\phi_y = 0$. Third, we focus on a special case with only two sectors. In sector 1, retailers set prices in a perfectly flexible way, while prices are sticky in sector 2. By using these assumptions, we can reduce the model to four endogenous equations and one monetary shock process¹². The endogenous equations include the equation for marginal costs, the Phillips curve, the Euler equation, and the Taylor rule.

3.5.1 Long-term relationship and history dependence

We begin with marginal costs¹³ in the model. As mentioned above, in models with the standard labour market, real marginal costs are equal to real wages. But within the search framework, real marginal costs consist of two components: real wages and hiring costs. This has been frequently pointed out by related literature, e.g. Krause and Lubik (2007). We denote the real hiring costs by $\hat{h}c_t$ ¹⁴. Rearranging the log-linearised version of Equation (3.8) obtains

¹²See Appendix for details.

¹³Retailers' real marginal costs are the real price of aggregate wholesale goods, and therefore a fixed-markup over firms' marginal costs: $mc_t = P_t^w/P_t = (\epsilon^w/(\epsilon^w-1))\varphi_t$. Therefore, when log-linearised, the marginal costs of retailers and firms are the same: $\hat{m}c_t = \hat{\varphi}_t$.

¹⁴For the full definition of the hiring costs, see Appendix.

$$\hat{m}c_t = \tilde{\varphi}_w \hat{w}_t + \tilde{\varphi}_{hc} \hat{h}c_t \quad (3.26)$$

where $\tilde{\varphi}_w = \tilde{w}/\tilde{\varphi}$ and $\tilde{\varphi}_{hc} = \tilde{h}c/\tilde{\varphi}$.

By using the definition of hiring costs together with Equations (3.4) and (3.6), and the production technologies of firms and retailers, we express hiring costs in terms of current, a lead, and a lag of output.

$$\hat{h}c_t = \varphi_1 \hat{y}_{t-1} + \varphi_2 \hat{y}_t + \varphi_3 E_t \hat{y}_{t+1} \quad (3.27)$$

$$\begin{aligned} \text{where } \varphi_1 &= -\varphi_\mu \varphi_\lambda / \lambda^2 \\ \varphi_2 &= \{\varphi_\mu (1 + (1 - \lambda) \varphi_\lambda) - \lambda(1 - \lambda)\} / \lambda^2 \\ \varphi_3 &= (1 - \lambda)(\lambda - \varphi_\mu) / \lambda^2 \\ \varphi_\lambda &= 1 - \lambda - p(\tilde{\theta}) \\ \varphi_\mu &= \mu / (1 - \mu) \end{aligned}$$

Finally, substituting Equation (3.27) into (3.26) yields an expression for the marginal costs in the models with the search frictions.

$$\hat{m}c_t = \varphi'_1 \hat{y}_{t-1} + \varphi'_2 \hat{y}_t + \varphi'_3 E_t \hat{y}_{t+1} \quad (3.28)$$

where $\varphi'_i = \tilde{\varphi}_{hc} \varphi_i$, $i = 1, 2, 3$. Equation (3.28) indicates that hiring costs, and therefore marginal costs negatively depend on lagged output. That is, real marginal costs are history dependent. We emphasize that this applies to any model with the search frictions, with or without nominal rigidities, with or without heterogeneity in nominal rigidities.

Before proceeding, we provide the intuition behind this history dependence. We begin by decomposing the partial derivative of the hiring costs with respect to the lagged output ($\partial \hat{h}c_t / \partial \hat{y}_{t-1}$), taking as given the current and future output levels (\hat{y}_t and \hat{y}_{t+1}), as follows.

$$\frac{\partial \hat{h}c_t}{\partial \hat{y}_{t-1}} = \frac{\partial p(\hat{\theta}_t)}{\partial \hat{y}_{t-1}} \cdot \frac{\partial \hat{\theta}_t}{\partial p(\hat{\theta}_t)} \cdot \frac{\partial \hat{h}c_t}{\partial \hat{\theta}_t} \quad (3.29)$$

The first component on the RHS is related to the fact that the equilibrium job-finding rate of unemployed workers can be lower when the lagged output level is

higher. This term arises due to the long-term nature of labour contracts under the search and matching framework. Consider the following log-linearised equations for employment and unemployment evolution.

$$\hat{n}_t = (1 - \lambda)\hat{n}_{t-1} + \lambda(p(\hat{\theta}_t) + \hat{u}_t) \quad (3.30)$$

$$\hat{u}_t = -\frac{p(\tilde{\theta})}{\lambda}\hat{n}_{t-1} \quad (3.31)$$

The first term of Equation (3.30) indicates the number of the existing workers that survived the job separation and the second term the number of the new matches. Therefore if we take the current workers (\hat{n}_t) as given, the higher the lagged employment (\hat{n}_{t-1}), the lower the demand for new hires. Equation (3.31) reflects the fact that more employment in the past leads to less unemployed job-seekers. Substituting Equation (3.31) into (3.30) yields

$$\hat{n}_t = \varphi_\lambda \hat{n}_{t-1} + \lambda p(\hat{\theta}_t) \quad (3.32)$$

$\varphi_\lambda = 1 - \lambda - p(\tilde{\theta})$ in Equation (3.32) implies the net effect of lagged employment on current employment. When the previous employment is higher, there are more surviving workers, by the fraction $1 - \lambda$. On the other hand, there are less job-seekers, and therefore the number of new matches produced is smaller, by the fraction $p(\tilde{\theta})$. If $\varphi_\lambda > 0$, then the equilibrium job-finding rate can be lower when previous employment is greater. Considering $\hat{n}_{t-1} = \hat{l}_{t-1} = \hat{y}_{t-1}$, this implies $\partial p(\hat{\theta}_t)/\partial \hat{y}_{t-1} = -\varphi_\lambda/\lambda(1-\mu)$.

The second component on the RHS in Equation (3.29) reflects the fact that the higher job-finding rate of workers require the higher labour market tightness: with the assumed functional form of the matching function, $p(\theta_t) = \mu_m \theta_t^{1-\mu}$. Therefore $\partial \hat{\theta}_t/\partial p(\hat{\theta}_t) = 1 - \mu$.

The last component on the RHS in Equation (3.29) arises from the fact that the vacancy-filling rate of firms is lower when the labour market tightness is higher. That is, $q(\theta_t) = \mu_m \theta_t^{-\mu}$. Tighter labour market means it takes longer to fill the vacancy, and therefore the average hiring costs per worker ($\kappa/q(\theta_t)$) are higher: $\partial \hat{h}_{ct}/\partial \hat{\theta}_t = \mu/\lambda$.

Combining all three components in Equation (3.29) gives us

$$\frac{\partial \hat{h}c_t}{\partial \hat{y}_{t-1}} = -\varphi_\mu \varphi_\lambda / \lambda^2 \quad (3.33)$$

It is trivial to check that $\partial \hat{h}c_t / \partial \hat{y}_{t-1}$ is equal to the coefficient on the lagged output in the hiring costs (Equation (3.27)).

3.5.2 Long-term employment relationship and heterogeneity in price stickiness

In this subsection, we combine five model equations to derive one reduced-form equation for output. First, under our assumption of heterogeneity in price stickiness, the Phillips curve¹⁵ is given by

$$\hat{\pi}_t = E_t \hat{\pi}_{t+1} + \frac{\alpha_1}{\alpha_2} E_t \hat{m}c_{t+1} + \{\kappa_{p2} + (2 + \kappa_{p2}) \frac{\alpha_1}{\alpha_2}\} \hat{m}c_t - \frac{\alpha_1}{\alpha_2} \hat{m}c_{t-1} \quad (3.34)$$

where $\kappa_{p2} = (1 - \delta_{p2})^2 / \delta_{p2}$.

By combining Equations (3.28) and (3.34) with the Euler equation, the Taylor rule, and the monetary shock process, we can express the output in terms of its own lags, leads, and the shock¹⁶.

$$\hat{y}_t = \tau_1 \hat{y}_{t-2} + \tau_2 \hat{y}_{t-1} + \tau_3 E_{t-1} \hat{y}_t + \tau_4 E_t \hat{y}_{t+1} + \tau_5 E_t \hat{y}_{t+2} + \tau_6 E_t \hat{y}_{t+3} + \tau_7 \hat{m}_t \quad (3.35)$$

where $\tau_1 = \phi_\pi \varphi'_1 \frac{\alpha_1}{\alpha_2} / \bar{\tau}_1$

$$\tau_2 = -\{\phi_\pi \bar{\tau}_2 \varphi'_1 + (\varphi'_1 - \phi_\pi \varphi'_2) \frac{\alpha_1}{\alpha_2}\} / \bar{\tau}_1$$

$$\tau_3 = \phi_\pi \varphi'_3 \frac{\alpha_1}{\alpha_2} / \bar{\tau}_1$$

$$\tau_4 = \{2 - \bar{\tau}_2 (\phi_\pi \varphi'_3 - \varphi'_2) - (\varphi'_1 + \varphi'_3 - \phi_\pi \varphi'_2) \frac{\alpha_1}{\alpha_2}\} / \bar{\tau}_1$$

$$\tau_5 = -\{1 - \bar{\tau}_2 \varphi'_3 + (\varphi'_2 - \phi_\pi \varphi'_3) \frac{\alpha_1}{\alpha_2}\} / \bar{\tau}_1$$

$$\tau_6 = -\varphi'_3 \frac{\alpha_1}{\alpha_2} / \bar{\tau}_1$$

$$\tau_7 = -(1 - \rho_m) / \bar{\tau}_1$$

¹⁵See Appendix for the second chapter for derivation.

¹⁶See Appendix for details.

$$\begin{aligned}\bar{\tau}_1 &= 1 + \bar{\tau}_2(\phi_\pi \varphi'_2 - \varphi'_1) - (\phi_\pi \varphi'_1 - \varphi'_2) \frac{\alpha_1}{\alpha_2} \\ \bar{\tau}_2 &= \kappa_{p2} + (2 + \kappa_{p2}) \frac{\alpha_1}{\alpha_2}\end{aligned}$$

In the second chapter, we report that, without the search frictions, the reduced-form output equation that is equivalent to Equation (3.35) has only one lag and two leads of output on the RHS along with the shock, i.e., $\tau_1 = \tau_3 = \tau_6 = 0$. Introducing the search frictions to the model augments the equation with one more lag and lead of output and the current output based on the expectation in the past ($E_{t-1}\hat{y}_t$). Note also that the search frictions (φ'_1 and φ'_3) and heterogeneity in price stickiness (α_1/α_2) interact in various ways and change every coefficient on the RHS of the equation. On the other hand, when we remove heterogeneity in price stickiness by setting $\alpha_1 = 0$ and $\kappa_{p2} = \kappa_p$, Equation (3.35) is reduced to the same form as in the second chapter, i.e., $\tau_1 = \tau_3 = \tau_6 = 0$ again. Lastly, when we set $\varphi'_1 = \varphi'_3 = \alpha_1/\alpha_2 = 0$ to remove any backward-lookingness from the search frictions and heterogeneity in price stickiness, all the lagged terms disappear, i.e. $\tau_1 = \tau_2 = \tau_3 = \tau_6 = 0$, and output becomes purely forward looking. Therefore Equation (3.35) suggests that the search frictions in the labour market and heterogeneity in nominal rigidity interact with each other, and this interaction can significantly affect the output dynamics.

Finally, we solve the model by the method of undetermined coefficients. We guess that y_t is a linear function of \hat{y}_{t-2} , \hat{y}_{t-1} , $E_{t-1}\hat{y}_t$, and \hat{m}_t .

$$\hat{y}_t = \chi_1 \hat{y}_{t-2} + \chi_2 \hat{y}_{t-1} + \chi_3 E_{t-1} \hat{y}_t + \chi_m \hat{m}_t \quad (3.36)$$

When we apply our guess, we obtain the following system of equations which we can solve for the coefficients in Equation (3.36).

$$\begin{aligned}\tau_6 \chi_1 \chi_2^3 + \tau_5 \chi_1 (1 - \chi_3) \chi_2^2 + \{\tau_4 \chi_1 (1 - \chi_3) + 2\tau_6 \chi_1^2\} (1 - \chi_3) \chi_2 \\ + (1 - \chi_3)^2 \{\tau_5 \chi_1^2 - \chi_1 (1 - \chi_3) + \tau_1 (1 - \chi_3)\} = 0\end{aligned} \quad (3.37)$$

$$\begin{aligned}
& \tau_6\chi_2^4 + \tau_5(1 - \chi_3)\chi_2^3 + \{\tau_4(1 - \chi_3)^2 + 2\tau_6\chi_1(1 - \chi_3) + \tau_6\chi_1\}\chi_2^2 \\
& + (1 - \chi_3)\{\tau_5(1 - \chi_3)\chi_1 + \tau_5\chi_1 - (1 - \chi_3)^2\}\chi_2 \\
& + (1 - \chi_3)\{\tau_6\chi_1^2 + \tau_4(1 - \chi_3)\chi_1 + \tau_2(1 - \chi_3)^2\} = 0
\end{aligned} \tag{3.38}$$

$$\begin{aligned}
& \tau_6\chi_3\chi_2^3 + \tau_5\chi_3(1 - \chi_3)\chi_2^2 + (1 - \chi_3)\chi_3\{\tau_4(1 - \chi_3) + 2\tau_6\chi_1\}\chi_2 \\
& + (1 - \chi_3)^2\{\tau_3(1 - \chi_3) + \tau_5\chi_1\chi_3 - \chi_3(1 - \chi_3)\} = 0
\end{aligned} \tag{3.39}$$

$$\begin{aligned}
& \chi_3[\tau_6\{\chi_2^3 + \rho_m\chi_2^2 + (1 - \chi_3)\chi_2\rho_m + (1 - \chi_3)\rho_m^3 + \\
& (1 - \chi_3)\chi_1(2\chi_2 + \rho_m)\} + \tau_5(1 - \chi_3)\{\chi_2^2 + \chi_2\rho_m + (1 - \chi_3)\rho_m \\
& + \chi_1(1 - \chi_3)\} + \tau_4(1 - \chi_3)^2\{\chi_2 + \rho_m - (1 - \chi_3)\}] + \tau_7(1 - \chi_3)^3 = 0
\end{aligned} \tag{3.40}$$

To further examine the influence of the long-term employment relationship and heterogeneity in nominal rigidity, we conduct two exercises¹⁷. We vary two key parameter values: φ_λ for the long-term labour contracts and α_1 for heterogeneity in price stickiness, and check (i) how the coefficients on the output equation change, and (ii) how the minimum monetary shock persistence required for an output hump is affected.

For the first experiment, it is useful to check the condition for a hump in output in terms of the coefficients on the output equation. The minimum requirement for a hump-shaped output response is $\hat{y}_{t+1} > \hat{y}_t$. Equation (3.36), together with $E_{t-1}\hat{y}_t = 0$, implies $\hat{y}_t = \chi_m\hat{m}_t$. Also, \hat{y}_{t+1} is given by $(1 - \chi_3)\hat{y}_{t+1} = \chi_2\hat{y}_t + \chi_m\hat{m}_{t+1}$. Given the monetary shock process and our assumption that $\epsilon_{t+1} = 0$, $(1 - \chi_3)\hat{y}_{t+1} = \chi_m(\chi_2 + \rho_m)\hat{m}_t$. Imposing $\hat{y}_{t+1} > \hat{y}_t$ gives us the condition for a hump in output to monetary shocks as follows.

$$\chi_2 + \chi_3 + \rho_m > 1 \tag{3.41}$$

This condition implies that when the sum $\chi_2 + \chi_3$ is large, along with sufficiently persistent shock, the model can generate a hump in output.

¹⁷Since the exact analytical solutions for the system of equations (3.37), (3.38), (3.39), and (3.40) involve solving a higher-order equation system, we rely on numerical solutions from now on.

Figure 3.3 reports how the coefficients (χ_1 , χ_2 , χ_3 , and χ_m) on the output equation change when we vary two parameter values. The first panel plots the coefficients against the long-term labour contracts (φ_λ). Apparently, the coefficient on one-period lagged output (χ_2) increases with the degree of long-term contracts. While χ_3 is negative in sign and decreases with φ_λ , the sum of the two increases as the degree of the long-term contracts increases. The second panel shows the coefficients for different values of the flexible-price sector share. The same pattern arises: χ_2 increases, χ_3 decreases, and the sum of the two increases with heterogeneity in price stickiness. This suggests that it is easier for the model to generate a hump in output to monetary shocks when labour contracts are based on longer-run relationship and when there is higher degree of heterogeneity in nominal rigidity.

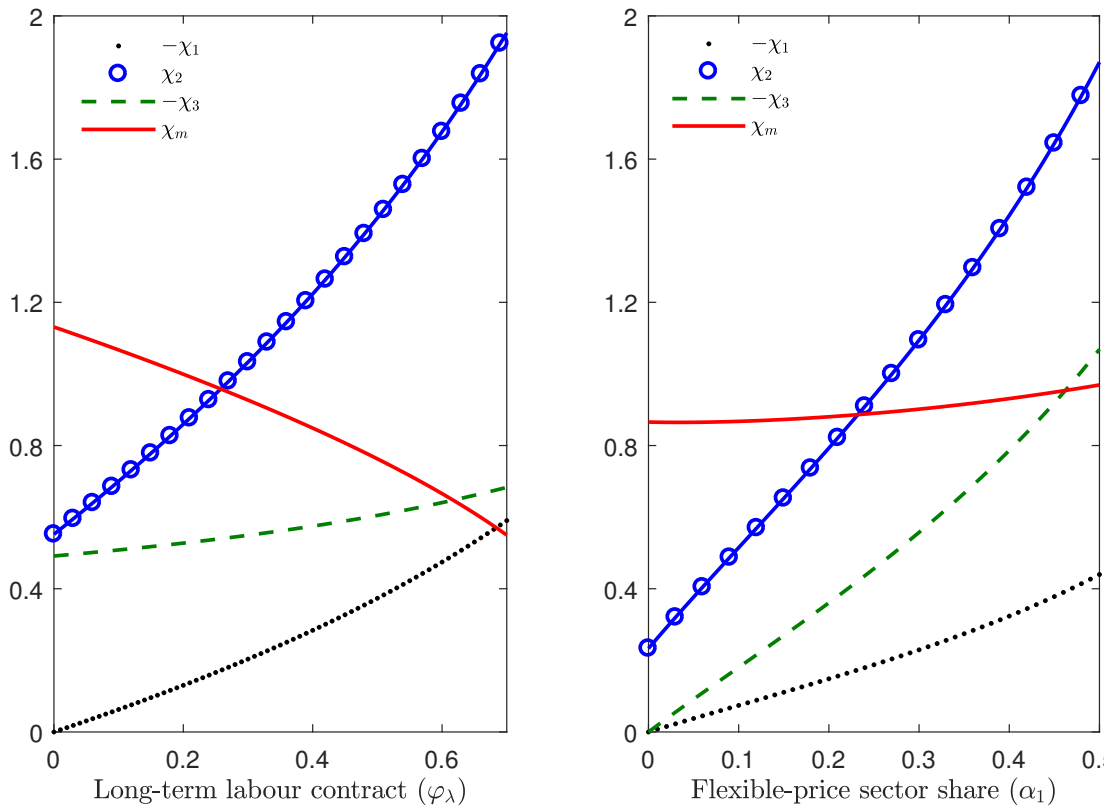


Figure 3.3: Long-term labour contracts, the flexible-price sector share, and the coefficients on the output equation

Note: The figure reports the coefficients on Equation (3.35) for different degrees of the long-term labour contracts (φ_λ) and heterogeneity in price stickiness (α_1). For the first panel, we set the flexible-price sector share to $\alpha_1 = 0.3$. For the second panel, we use the standard calibration for the long-term contracts: $\varphi_\lambda = 0.3$.

Next, we measure the potential of the long-term contracts and heterogeneity in nominal rigidity for an output hump in a more succinct way. Since more persistent shock leads to a nicer hump, and the shock persistence (ρ_m) is independent of the coefficients on output lags, the minimum required shock persistence for a hump in output fits our purpose well. In other words, if a model requires only lower degree of monetary shock persistence, we interpret this as indicating greater potential of the model for a hump-shaped output response. Figure 3.4 shows the minimum required shock persistence for the second-period peak and the third-period peak of output responses to monetary shocks. We calculate the minimum shock persistence by simulating the IRF of output and checking at what value of ρ_m the conditions $\hat{y}_t < \hat{y}_{t+1}$ and $\hat{y}_{t+1} < \hat{y}_{t+2}$ are met. As shown in the first panel, the long-term nature of contracts significantly reduces the need for a highly persistent shock. For example, when $\varphi_\lambda = 0.1$, the shock persistence must be greater than 0.81, for output response to have a peak at the second period. The persistence must be at least 0.99, for the third-period output peak. When $\varphi_\lambda = 0.5$, the persistence can be as low as 0.17 for the second-period peak and 0.78 for the third-period peak. The same holds true for heterogeneity in price stickiness, as shown in the second panel. When $\alpha_1 = 0.1$, the shock persistence must be greater than 0.67 and 0.95, for the second-period and the third-period output peaks. When $\alpha_1 = 0.5$, it can be lower at 0.20 and 0.84, respectively. Clearly, the potential of the model to generate a hump-shaped output response to monetary shocks is greater when the labour contracts are based on longer-run relationship and when there is greater heterogeneity in nominal rigidity.

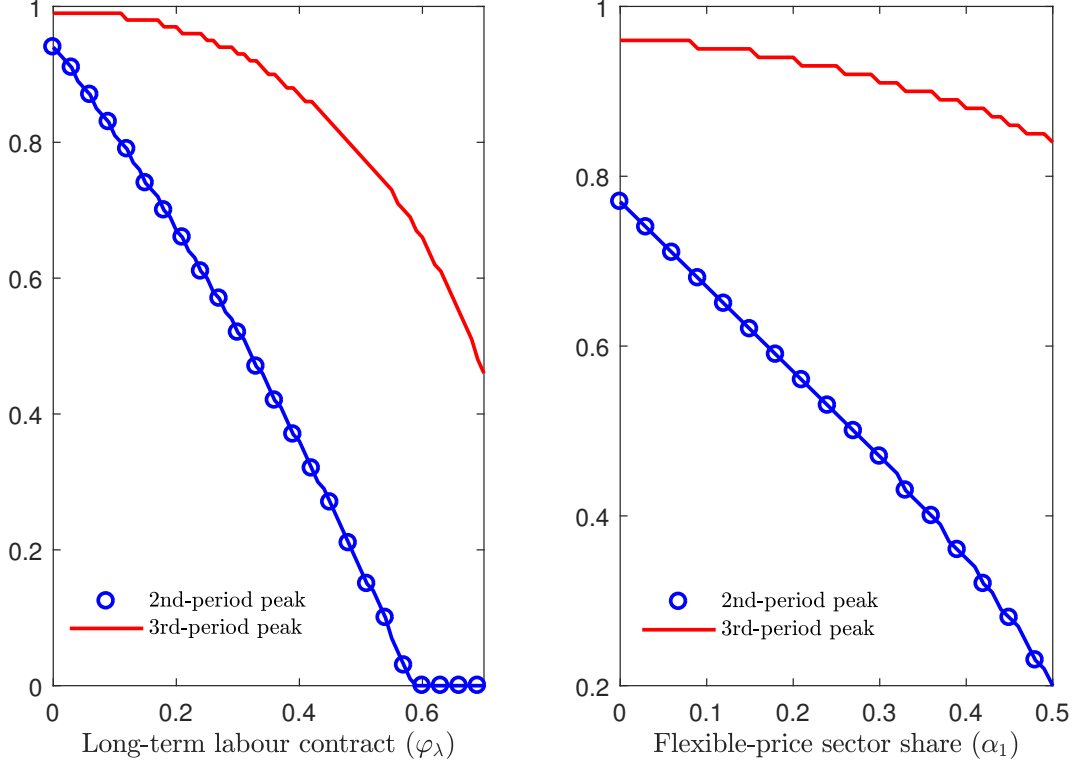


Figure 3.4: Long-term labour contracts, the flexible-price sector share, and the minimum required shock persistence

Note: The minimum shock persistences for the second-period peak and the third-period peak are calculated by simulating the IRF of output and checking at what value of ρ_m the conditions $\hat{y}_t < \hat{y}_{t+1}$ and $\hat{y}_{t+1} < \hat{y}_{t+2}$ are met.

Lastly, we examine how the long-term contracts and heterogeneity in nominal rigidity interact. Figure 3.5 shows how this interaction affects the minimum required monetary shock persistence. The first panel plots the required shock persistence for an output hump (the second-period peak) against the long-term labour contracts (φ_λ), for two values of the flexible-price sector share ($\alpha_1 = 0.0$ and $\alpha_1 = 0.3$). In both cases, the long-term contracts reduce the required shock persistence, but the decrease is much steeper when there is heterogeneity in price stickiness, that is, when $\alpha_1 = 0.3$. The second panel plots the required shock persistence against the flexible-price sector share (α_1), for two values of the long-term contracts ($\varphi_\lambda = 0.0$ and $\varphi_\lambda = 0.3$). When $\varphi_\lambda = 0.3$, the decrease in the required shock persistence is very clear, while the decrease is modest when $\varphi_\lambda = 0.0$. We conclude that the long-term labour contracts and heterogeneity in nominal rigidity reinforce each other to help the model generate a hump-shaped output response to monetary shocks.

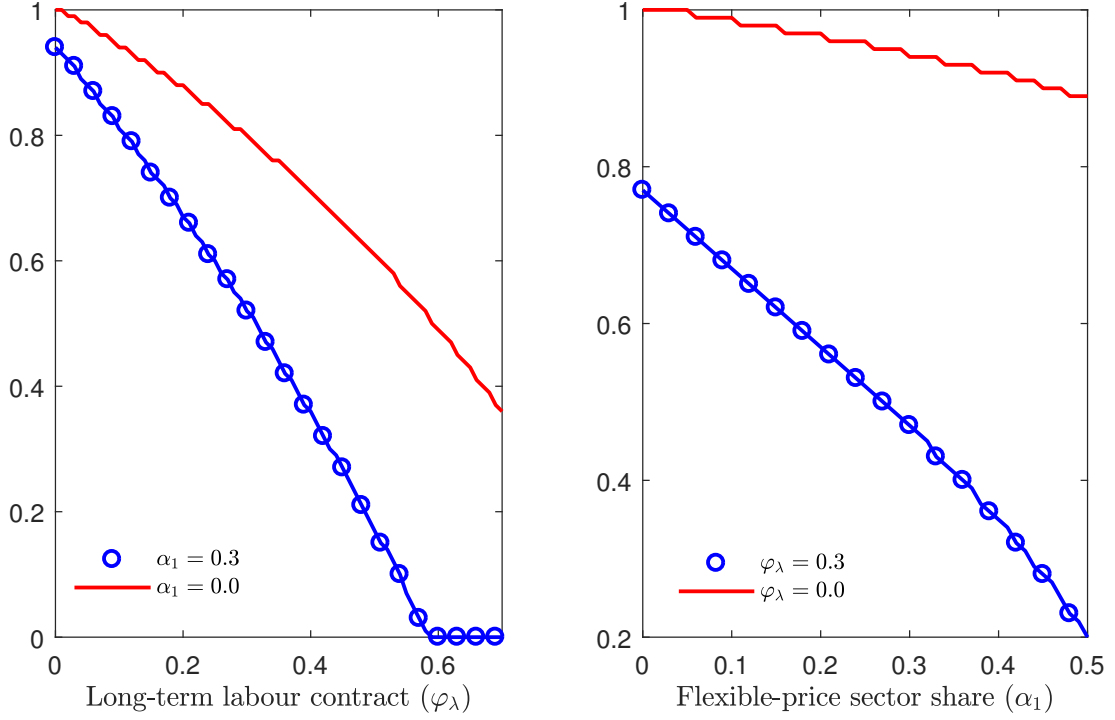


Figure 3.5: An interaction between the long-term labour contracts and the flexible-price sector share on the minimum required shock persistence

Note: The minimum shock persistence for an output hump is calculated by simulating the IRF of output and checking at what value of ρ_m the condition $\hat{y}_t < \hat{y}_{t+1}$ is met.

3.6 Summary and conclusion

This chapter has revealed a link between the search and matching frictions and heterogeneity in nominal rigidities. The long-term nature of employment relationship that arises under the search and matching framework makes marginal costs history dependent. This, when combined with heterogeneity in nominal rigidities, can generate inertia in output dynamics, leading to a hump-shaped output response if a monetary shock is persistent.

Since the interaction between the search frictions and heterogeneity in nominal rigidities can significantly affect inflation and output dynamics in the New Keynesian models, it is worthwhile to explore its implication in larger-scale models. It will deepen our understanding of the model to see how the inertia that arises from long-term employment relationship and heterogeneity in nominal rigidities can interact with other components of marginal costs in such models, e.g. capital adjustment

costs and intermediate input prices. In addition, it will be fruitful to examine whether and how the conclusions on the optimal monetary policy are altered by our findings.

Concluding Remarks

We have developed a framework that sheds light on the functioning of the labour market and its implication for output dynamics and macroeconomy. In the first chapter, we build a RBC model with the search frictions in the labour market and heterogeneity in wage stickiness. We show that models that account for heterogeneity can match both the observed volatility in unemployment and the cyclicalities of wages, while the models without heterogeneity exhibit too low wage cyclicalities. The key to our results is the presence of flexible wages in the economy. Since flexible wages respond more to shocks than sticky wages, average wages in our multi-sector models are more sensitive to shocks than their one-sector counterparts.

The second and the third chapters provide an alternative explanation for a key feature of monetary-policy transmission mechanism, that output shows a hump-shaped response to monetary shocks. The second chapter focuses on the role of heterogeneity in price stickiness for output dynamics. We present a simple New Keynesian model with the standard labour market and with heterogeneity in price stickiness. We formally demonstrate how the presence of flexible prices makes price-setting history dependent. As a result, inflation depends on lagged output. This history dependence, in the presence of persistent monetary policy shocks, leads to a hump-shaped response in output.

In the third chapter, we bring together our findings in the first and the second chapters. Our aim is to revisit the role of the search and matching frictions for output dynamics in New Keynesian business cycle models, in combination with heterogeneity in nominal rigidities. We bring the RBC model that we developed in the first chapter into New Keynesian environment, and introduce heterogeneity in price stickiness. Our key finding is that long-term employment relationship that

arises under search and matching framework plays a similar role as heterogeneity in price stickiness does in the second chapter. It makes marginal costs history dependent: marginal costs depend on lagged output level. Inflation depends on marginal costs, and on lagged output level. Therefore previous path of output becomes relevant.

In the second and the third chapters, we develop a novel method to summarise the model into one reduced-form equation for output. By using this method, we analytically show that output exhibits an inertial response in models with the search frictions and/or heterogeneity in nominal rigidities. The inertia becomes stronger when there is higher degree of heterogeneity and when labour contracts are based on longer-term relationship. We also show that the search frictions and heterogeneity in nominal rigidities reinforce each other to yield a mechanism that generates inertia in the model.

This dissertation has shown that interactions between a realistic description for the labour market and that for price/wage setting can help us address important challenges in macroeconomics. Given the importance of these interactions for output and inflation dynamics, we suggest that estimating our model in medium-sized New Keynesian models would be a promising avenue for future research.

Appendix for Chapter 1

In this Appendix, we present the steady-state of the model economy. The variables with a tilde denote the steady-state values. Note that the steady-state is the same across different models. Note also that in the steady state, sectoral wage (\tilde{w}_i) and hiring rate (\tilde{x}_i) are the same across sectors.

Consumption and savings:

$$1 = \beta(1 + \tilde{r})$$

Production:

$$\tilde{y} = \tilde{A}\tilde{n}$$

Separation and hiring rate:

$$\tilde{x} = \lambda$$

Job creation:

$$\kappa\tilde{x} = \tilde{A} - \tilde{w} + \beta\left\{\frac{\kappa}{2}\tilde{x}^2 + (1 - \lambda)\kappa\tilde{x}\right\}$$

Wage:

$$\tilde{w} = \eta\left\{\tilde{A} + \beta\frac{\kappa}{2}\tilde{x}^2 + \beta p(\tilde{\theta})\kappa\tilde{x}\right\} + (1 - \eta)b$$

Flows in and out of unemployment:

$$\tilde{x}(1 - \tilde{u}) = p(\tilde{\theta})\tilde{u}$$

Search and matching:

$$p(\tilde{\theta})\tilde{u} = \sigma_m \tilde{u}^\mu \tilde{v}^{1-\mu}$$

Resource constraint:

$$1 = \frac{\tilde{c}}{\tilde{y}} + \frac{\kappa}{2} \tilde{x}^2 \frac{\tilde{n}}{\tilde{y}}$$

Appendix for Chapter 2

In this Appendix, we present the derivation of the output equation (Equation (2.13) for the MC). We first deal with the case in which $\beta = 1$ and $\phi_y = 0$, which is presented in Section 3 of Chapter 2 of the main text. We also present the output equation for a more general case when $\beta \neq 1$ and $\phi_y \neq 0$.

Phillips curve in sector 1 and 2 are given, respectively, by

$$\pi_{1t} = \gamma(y_t - y_{t-1}) + \pi_t \quad (\text{B.1})$$

$$\pi_{2t} = E_t \pi_{2t+1} + \kappa_2(\gamma y_t - p_{2t}) \quad (\text{B.2})$$

Substituting Equation (B.2) into aggregate inflation $\pi_t = \alpha_1 \pi_{1t} + \alpha_2 \pi_{2t}$ and rearranging yield

$$\pi_t = \pi_{2t} + \frac{\alpha_1}{\alpha_2} \gamma(y_t - y_{t-1}) \quad (\text{B.3})$$

Solving Equation (B.3) for π_{2t} , bring it forward by one period to obtain π_{2t+1} , and taking expectations at time t yield

$$E_t \pi_{2t+1} = E_t \pi_{t+1} - \frac{\alpha_1}{\alpha_2} \gamma(E_t y_{t+1} - y_t) \quad (\text{B.4})$$

By substituting $E_t \pi_{2t+1}$ in Equation (B.4) into Equation (B.2) and making use of Equations (2.4) and (2.8), we obtain

$$\pi_{2t} = E_t \pi_{t+1} - \frac{\alpha_1}{\alpha_2} \gamma(E_t y_{t+1} - y_t) + \kappa_2(1 + \frac{\alpha_1}{\alpha_2}) \gamma y_t \quad (\text{B.5})$$

When we substitute Equation (B.5) into (B.3), we obtain the Phillips curve in the

MC. This corresponds to Equation (2.12) in the main text.

$$\pi_t = E_t \pi_{t+1} - \frac{\alpha_1}{\alpha_2} \gamma E_t y_{t+1} + \bar{\tau}_2 y_t - \frac{\alpha_1}{\alpha_2} \gamma y_{t-1} \quad (\text{B.6})$$

where $\bar{\tau}_2 \equiv \{\kappa_2 + (2 + \kappa_2)\alpha_1/\alpha_2\}\gamma$. Next, by combining the Euler equation and the Taylor rule, we obtain

$$y_t - E_t y_{t+1} + \phi_\pi \pi_t + m_t - E_t \pi_{t+1} = 0 \quad (\text{B.7})$$

We solve Equation (B.6) for $E_t \pi_{t+1}$, substitute it into Equation (B.7), and solve the resulting equation for π_t to obtain

$$\pi_t = \frac{1}{1 - \phi_\pi} \left[-\left(1 + \frac{\alpha_1}{\alpha_2}\right) E_t y_{t+1} + (1 + \bar{\tau}_2) y_t - \frac{\alpha_1}{\alpha_2} \gamma y_{t-1} + m_t \right] \quad (\text{B.8})$$

We bring Equation (B.8) forward by one period to obtain π_{t+1} , take expectations at time t , and substitute the resulting expressions for π_t and $E_t \pi_{t+1}$ into Equation (B.7), making use of $E_t m_{t+1} = \rho_m m_t$, to obtain Equation (2.13) in the main text.

Finally, we present the output equation for a more general case when $\beta \neq 1$ and $\phi_y \neq 0$.

$$\begin{aligned} y_t &= \tau_1 y_{t-1} + \tau_2 E_t y_{t+1} + \tau_3 E_t y_{t+2} + \tau_4 m_t \\ \text{where } \tau_1 &= \phi_\pi \frac{\alpha_1}{\alpha_2} \gamma / \bar{\tau}_1 \\ \tau_2 &= (2\beta + \beta\phi_y + \beta\phi_\pi \frac{\alpha_1}{\alpha_2} \gamma + \bar{\tau}_2) / \bar{\tau}_1 \\ \tau_3 &= -\beta(1 + \frac{\alpha_1}{\alpha_2} \gamma) / \bar{\tau}_1 \\ \tau_4 &= -(1 - \beta\rho_m) / \bar{\tau}_1 \\ \bar{\tau}_1 &= 1 + \phi_y + \phi_\pi \bar{\tau}_2 + \frac{\alpha_1}{\alpha_2} \gamma \\ \bar{\tau}_2 &= \{\kappa_2 + (1 + \beta + \kappa_2) \frac{\alpha_1}{\alpha_2}\} \gamma \end{aligned}$$

Appendix for Chapter 3

The definition of the hiring costs

A straight forward manipulation of the Equation (3.8) obtains

$$\underbrace{\varphi_t}_{\text{Marginal costs}} = \underbrace{w_t}_{\text{Real wage}} + \underbrace{\frac{\kappa}{q(\theta_t)} - (1-\lambda)\beta \frac{\kappa}{E_t q(\theta_{t+1})}}_{\text{Hiring costs}} \quad (\text{C.1})$$

We define the sum of the second and the third terms on the RHS of Equation (C.1) as hiring costs. The first term, $\frac{\kappa}{q(\theta_t)}$, denotes the current (average) cost of hiring one additional worker. The second term, $-(1-\lambda)\beta \frac{\kappa}{E_t q(\theta_{t+1})}$, indicates that the discounted continuation value of a surviving worker should be subtracted from the current costs.

A reduced-form model

Under the assumptions made in Section 4 of Chapter 3, the model can be represented by the following log-linearised equation system.

Employment

$$\hat{n}_t = (1-\lambda)\hat{n}_{t-1} + \lambda\{p(\hat{\theta}_t) + \hat{u}_t\} \quad (\text{C.2})$$

Unemployment

$$\hat{u}_t = -\frac{p(\tilde{\theta})}{\lambda}\hat{n}_{t-1} \quad (\text{C.3})$$

Job creation condition

$$\frac{\kappa}{q(\tilde{\theta})}\mu\hat{\theta}_t = \tilde{\varphi}\hat{\varphi}_t - \tilde{w}\hat{w}_t + (1-\lambda)\frac{\kappa}{q(\tilde{\theta})}(E_t\hat{\Lambda}_{t,t+1} - \mu E_t\hat{\theta}_{t+1}) \quad (\text{C.4})$$

Production

$$\hat{y}_t = \hat{l}_t = \hat{n}_t \quad (\text{C.5})$$

Resource constraint

$$\hat{y}_t = \hat{c}_t \quad (\text{C.6})$$

Marginal costs of firms and retailers

$$\hat{m}c_t = \hat{\varphi}_t \quad (\text{C.7})$$

Relative price of sector 1

$$\hat{p}_{1t} = \hat{m}c_t \quad (\text{C.8})$$

Phillips curve in sector 2

$$\hat{\pi}_{2t} = E_t \hat{\pi}_{2t+1} + \kappa_{p2}(\hat{m}c_t - \hat{p}_{2t}) \quad (\text{C.9})$$

Sectoral inflation

$$\hat{p}_{it} = \hat{p}_{it-1} + \hat{\pi}_{it} - \hat{\pi}_t, \quad i = 1, 2 \quad (\text{C.10})$$

Aggregate inflation

$$\hat{\pi}_t = \alpha_1 \hat{\pi}_{1t} + \alpha_2 \hat{\pi}_{2t} \quad (\text{C.11})$$

Euler equation

$$\hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1}) \quad (\text{C.12})$$

Taylor rule

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \hat{m}_t \quad (\text{C.13})$$

Monetary shock process

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_t^m \quad (\text{C.14})$$

By combining Equations (C.2), (C.3), (C.5), and (C.6), together with $p(\hat{\theta}_t) =$

$(1 - \mu)\hat{\theta}_t$, we obtain

$$\hat{\theta}_t = \frac{1}{\lambda(1 - \mu)}(\hat{y}_t - \varphi_\lambda \hat{y}_{t-1}) \quad (\text{C.15})$$

By combining Equation (C.4) and (C.7), together with $\hat{w}_t = 0$ and $\hat{\Lambda}_{t,t+1} = -\hat{c}_{t+1} + \hat{c}_t$ gives

$$\hat{m}c_t = \tilde{\varphi}_{hc} \hat{h}c_t \quad (\text{C.16})$$

where

$$\hat{h}c_t = \frac{\mu}{\lambda} \{ \hat{\theta}_t - (1 - \lambda)E_t \hat{\theta}_{t+1} \} + (1 - \lambda)(E_t \hat{y}_{t+1} - \hat{y}_t) \quad (\text{C.17})$$

Substituting Equations (C.15) and (C.17) into (C.16) obtains the following expression for the marginal costs of retailers.

$$\hat{m}c_t = \varphi'_1 \hat{y}_{t-1} + \varphi'_2 \hat{y}_t + \varphi'_3 E_t \hat{y}_{t+1} \quad (\text{C.18})$$

This corresponds to Equation (3.28) in the text.

Next, by combining Equations (C.8), (C.9), (C.10), and (C.11), we obtain the following aggregate Phillips curve.

$$\hat{\pi}_t = E_t \hat{\pi}_{t+1} + \frac{\alpha_1}{\alpha_2} E_t \hat{m}c_{t+1} + \{ \kappa_{p2} + (2 + \kappa_{p2}) \frac{\alpha_1}{\alpha_2} \} \hat{m}c_t - \frac{\alpha_1}{\alpha_2} \hat{m}c_{t-1} \quad (\text{C.19})$$

Substituting Equation (C.6) into (C.12) gives

$$\hat{y}_t = E_t \hat{y}_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1}) \quad (\text{C.20})$$

Consequently we have a five-equation reduced form of the model that includes Equations (C.18), (C.19), (C.20), (C.13), and (C.14).

The derivation of Equation (3.35)

First, we combine the Euler equation and the Taylor rule to obtain

$$\hat{y}_t = E_t \hat{y}_{t+1} + \phi_\pi \hat{\pi}_t + \hat{m}_t - E_t \hat{\pi}_{t+1} \quad (\text{C.21})$$

We substitute the marginal costs (Equation (C.18)) into the aggregate Phillips curve (Equation (C.19)) to obtain

$$\begin{aligned}\hat{\pi}_t = & E_t \hat{\pi}_{t+1} - \frac{\alpha_1}{\alpha_2} \varphi'_1 \hat{y}_{t-2} + (\bar{\tau}_2 \varphi'_1 - \frac{\alpha_1}{\alpha_2} \varphi'_2) \hat{y}_{t-1} - \frac{\alpha_1}{\alpha_2} \varphi'_3 E_{t-1} \hat{y}_t \\ & - (\frac{\alpha_1}{\alpha_2} \varphi'_1 - \bar{\tau}_2 \varphi'_2) \hat{y}_t - (\frac{\alpha_1}{\alpha_2} \varphi'_2 - \bar{\tau}_2 \varphi'_3) E_t \hat{y}_{t+1} - \frac{\alpha_1}{\alpha_2} \varphi'_3 E_t \hat{y}_{t+2}\end{aligned}\quad (\text{C.22})$$

By solving Equation (C.22) for $E_t \hat{\pi}_{t+1}$, substituting it into Equation (C.21), and solving the resulting equation for $\hat{\pi}_t$, we obtain

$$\begin{aligned}\hat{\pi}_t = & \frac{1}{\phi_\pi - 1} \left[\frac{\alpha_1}{\alpha_2} \varphi'_1 \hat{y}_{t-2} + (\frac{\alpha_1}{\alpha_2} \varphi'_2 - \bar{\tau}_2 \varphi'_1) \hat{y}_{t-1} + \frac{\alpha_1}{\alpha_2} \varphi'_3 E_{t-1} \hat{y}_t \right. \\ & \left. + (\frac{\alpha_1}{\alpha_2} \varphi'_1 - \bar{\tau}_2 \varphi'_2 - 1) \hat{y}_t + (\frac{\alpha_1}{\alpha_2} \varphi'_2 - \bar{\tau}_2 \varphi'_3 + 1) E_t \hat{y}_{t+1} + \frac{\alpha_1}{\alpha_2} \varphi'_3 E_t \hat{y}_{t+2} \right]\end{aligned}\quad (\text{C.23})$$

We bring Equation (C.23) forward to obtain $E_t \hat{\pi}_{t+1}$, and substitute $\hat{\pi}_t$, $E_t \hat{\pi}_{t+1}$, and Equation (C.14) into (C.21) to obtain Equation (3.35) in the main text.

Bibliography

- Ascari, Guido (2000). “Optimising Agents, Staggered Wages and the Persistence in the Real Effects of Money Shocks”. In: *The Economic Journal* 110, pp. 664–686.
- Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014). “Some Evidence on the Importance of Sticky Wages”. In: *American Economic Journal: Macroeconomics* 6, pp. 70–101.
- Barro, Robert J. (1977). “Long-term Contracting, Sticky Prices, and Monetary Policy”. In: *Journal of Monetary Economics* 3.3, pp. 305–316.
- Basu, Susanto and C. L. House (2016). *Allocative and Remitted Wages: New Facts and Challenges for Keynesian Models in: J.B. Taylor and Harald Uhlig, eds, Handbook of Macroeconomics*. Vol. 2. North-Holland, Amsterdam, pp. 297–354.
- Bernanke, Ben S and Alan S. Blinder (1992). “The Federal Funds Rate and the Channels of Monetary Transmission”. In: *American Economic Review* 82, pp. 901–921.
- Bils, Mark and Pete Klenow (2004). “Some Evidence on the Importance of Sticky Prices”. In: *Journal of Political Economy* 112.5, pp. 947–985.
- Bils, Mark, Peter Klenow, and Benjamin Malin (2012). “Reset Price Inflation and the Impact of Monetary Policy Shocks”. In: *American Economic Review* 102.6, pp. 2798–2825.
- Blanchard, Olivier and Jordi Gali (2010). “Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment”. In: *American Economic Journal: Macroeconomics* 2.2, pp. 1–30.

- Calvo, Guillermo A. (1983). “Staggered prices in a utility-maximizing framework”. In: *Journal of Monetary Economics* 12.3, pp. 383–398.
- Carvalho, Carlos (2006). “Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks”. In: *Frontiers of Macroeconomics* 2.1.
- Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan (2009). “New Keynesian Models: Not Yet Useful for Policy Analysis”. In: *American Economic Journal: Macroeconomics* 1.1, pp. 242–66.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (1996). “The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds”. In: *The Review of Economics and Statistics* 78, pp. 16–34.
- (1999). *Monetary Policy Shocks: What Have We Learned and to What End?* Vol. 1. North-Holland, Amsterdam, pp. 65–148.
- (2005). “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”. In: *Journal of Political Economy* 113.1, pp. 1–45.
- Christiano, Lawrence J., Martin Eichenbaum, and Mathias Trabandt (2017). “On DSGE Models”. In: *Unpublished manuscript*.
- Costain, James S. and Michael Reiter (2008). “Business Cycles, Unemployment Insurance, and the Calibration of Matching Models”. In: *Journal of Economic Dynamics and Control* 32, pp. 1120–1155.
- Davis, Steven J., John C. Haltiwanger, and Scott Schuh (1996). *Job Creation and Destruction*. MIT Press, Cambridge, MA.
- Den Haan, Wouter J., Garey Ramey, and Joel Watson (2000). “Job destruction and Propagation of shocks”. In: *American Economic Review* 90.3, pp. 482–498.
- Diamond, Peter A. (1982a). “Aggregate Demand Management in Search Equilibrium”. In: *Journal of Political economy* 90.5, pp. 881–894.
- (1982b). “Wage Determination and Efficiency in Search Equilibrium”. In: *Review of Economic Studies* 49.2, pp. 217–227.
- Dixon, Huw and Engin Kara (2006). “How to compare Taylor and Calvo contracts: a comment on Michael Kiley”. In: *Journal of Money, Credit and Banking* 38.2.

- Dixon, Huw and Engin Kara (2010). “Can we explain inflation persistence in a way that is consistent with the micro-evidence on nominal rigidity?” In: *Journal of Money, Credit and Banking* 42.1, pp. 151–170.
- (2010a). “Contract Length Heterogeneity and the Persistence of Monetary Shocks in a Dynamic Generalized Taylor Economy”. In: *European Economic Review* 55.5.
- Dixon, Huw and Herve Le Bihan (2012). “Generalized Taylor and Generalized Calvo Price and Wage-Setting: Micro Evidence with Macro Implications”. In: *Economic Journal* (forthcoming).
- Fuhrer, Jeffrey (2000). “Habit Formation in Consumption and Its Implications for Monetary Policy”. In: *American Economic Review* 90.3, pp. 367–390.
- Galí, Jordi (2015). *Monetary Policy, Inflation, and the Business Cycle*. 2nd ed. Princeton University Press, pp. 64–66.
- Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari (2016). “Unemployment Fluctuations, Match Quality, and the Wage Cyclicalities of New Hires”. In: *NBER Working Paper* 22341.
- Gertler, Mark, Luca Sala, and Antonella Trigari (2008). “An estimated monetary DSGE model with unemployment and staggered Nominal wage bargaining”. In: *Journal of Money, Credit, and Banking* 40.8, pp. 1713–1764.
- Gertler, Mark and Antonella Trigari (2009). “Unemployment Fluctuations with Staggered Nash Wage Bargaining”. In: *Journal of Political economy* 117.1, pp. 38–86.
- Haefke, Christian, Marcus Sonntag, and Thijs Van Rens (2013). “Wage Rigidity and Job Creation”. In: *Journal of Monetary Economics* 60, pp. 887–899.
- Hagedorn, Marcus and Iourii Manovskii (2013). “Job selection and Wages over the Business Cycle”. In: *American Economic Review* 103, pp. 771–803.
- Hall, Robert E. (2005). “Employment Fluctuations with Equilibrium Wage Stickiness”. In: *American Economic Review* 95.1, pp. 50–65.
- (2008). “Sources and mechanisms of cyclical fluctuations in the labor market”. In: *Mimeo*.
- Hume, David (1752). *On Money in Essays*. George Routledge and Sons, London.

- Kara, Engin (2015). “The reset inflation puzzle and the heterogeneity in price stickiness”. In: *Journal of Monetary Economics* 76, pp. 29–37.
- Kennan, John (2010). “Private information, wage bargaining and employment fluctuations”. In: *The Review of Economic Studies* 77, pp. 633–664.
- Keynes, John Maynard (1936). *General Theory of Employment, Interest, and Money*. Prometheus Books, New York.
- Krause, Michael U. and Thomas A. Lubik (2007). “The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions”. In: *Journal of Monetary Economics* 54, pp. 706–727.
- Kudlyak, Marianna (2014). “The cyclical cost of the user cost of labor”. In: *Journal of Monetary Economics* 68.
- Lucas, Robert E. (1976). “Econometric Policy Evaluation: A critique”. In: *Carnegie-Rochester Conference Series on Public Policy* 1, pp. 19–46.
- Menzio, Guido (2005). “High-frequency wage rigidity”. In: *University of Pennsylvania, unpublished manuscript*.
- Menzio, Guido and Espen R. Moen (2010). “Worker replacement”. In: *Journal of Monetary Economics* 57, pp. 623–636.
- Merz, Monika (1995). “Search in Labor Market and the Real Business Cycle”. In: *Journal of Monetary Economics* 36, pp. 269–300.
- Mortensen, Dale T. (1982). “Property Rights and Efficiency in Mating, Racing, and Related Games”. In: *American Economic Review* 72.5, pp. 968–979.
- Mortensen, Dale T. and Christopher A. Pissarides (1994). “Job Creation and Job Destruction in the Theory of Unemployment”. In: *Review of Economic Studies* 61.3, pp. 397–415.
- Nakamura, Emi and Jon Steinsson (2008). “Five Facts About Prices: A Reevaluation of Menu Cost Models”. In: *Quarterly Journal of Economics* 123.4, pp. 1415–1464.
- Pigou, Arthur C. (1968). *The Theory of Unemployment*. Frank Cass and Co., London.

- Pissarides, Christopher A. (1985). “Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages”. In: *American Economic Review* 75.4, pp. 676–690.
- (2000). *Equilibrium Unemployment Theory*. The MIT Press, Cambridge.
- (2009). “The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?” In: *Econometrica* 77.5, pp. 1339–1369.
- Ravenna, Federico and Carl E. Walsh (2008). “Vacancies, Unemployment, and the Phillips curve”. In: *European Economic Review* 52.8, pp. 1494–1521.
- Ricardo, David (1817). *On the Principles of Political Economy and Taxation*. John Murray, London.
- Rotemberg, Julio J. and Michael Woodford (1997). “An Optimization-based Econometric Framework for the Evaluation of Monetary Policy in: Ben S. Bernanke and Julio J. Rotemberg eds., NBER macroeconomic annual: 1997”. In: pp. 297–346.
- Shimer, Robert (2005). “The Cyclical Behavior of Equilibrium Unemployment and Vacancies”. In: *American Economic Review* 95.1, pp. 25–49.
- Sims, Christopher A. (1986). “Are Forecasting Models Usable for Policy Analysis?” In: *Federal Reserve Bank of Minneapolis Quarterly Review* 10, pp. 2–16.
- Smets, Frank and Rafael Wouters (2007). “Shocks and Frictions in US Business Cycles: A Bayesian DSGE approach”. In: *American Economic Review* 97.32, pp. 586–607.
- Stuber, Heiko (2017). “The Real Wage Cyclicalities of Newly Hired and Incumbent Workers in Germany”. In: *The Economic Journal* 127, pp. 522–546.
- Taylor, John B. (2016). *The Staying Power of Staggered Wage and Price Setting Models in Macroeconomics in: John B. Taylor and Harald Uhlig, eds, Handbook of Macroeconomics*. Vol. 2. North-Holland, Amsterdam, pp. 2009–2042.
- Thomas, Carlos (2008). “Search and Matching Frictions and Optimal Monetary Policy”. In: *Journal of Monetary Economics* 55, pp. 936–956.
- Trigari, Antonella (2009). “Equilibrium Unemployment, Job Flows, and Inflation Dynamics”. In: *Journal of Money, Credit, and Banking* 41, pp. 1–33.

- Walsh, Carl (2005). “Endogenous objectives and the evaluation of targeting rules for monetary policy”. In: *Journal of Monetary Economics* 52, pp. 889–911.
- Woodford, Michael (2003). “Interest and Prices: Foundations of a Theory of Monetary Policy”. In: *Princeton University Press, Princeton, NJ*.